

# Unit 6 - Day 1

## Geo X Angles of a Polygon: Investigation

Name Key - 2015

**Part 1** Use the reference sheet given with the diagram of polygons divided into triangles. Complete the chart below using the diagrams. Can you find the pattern for the interior angle sum (the sum of all of the angles in the polygon)?

Polygon	Number of sides	Number of diagonals from 1 vertex	Number of triangles	Interior angle sum
Triangle	3	0	1 $\times 180 =$	180°
Quadrilateral	4	1	2 $\times 180 =$	360°
Pentagon	5	2	3 $\times 180 =$	540°
Hexagon	6	3	4 $\times 180 =$	720°
Heptagon (Septagon)	7	4	5 $\times 180 =$	900°
Octagon	8	5	6 $\times 180 =$	1080°
Nonagon	9	6	7 $\times 180 =$	1260°
Decagon	10	7	8 $\times 180 =$	1440°
Dodecagon	12	9	10 $\times 180 =$	1800°
⋮				
16-gon (Hexdecagon)	16	13	14 $\times 180 =$	2520°

1. What pattern do you notice for the interior angle sum of the polygons in the table?

As the number of sides increases by one, the sum increases by 180° for each polygon.

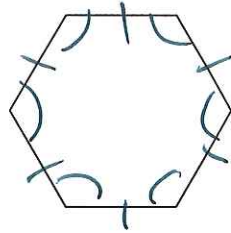
2. What do you notice about the relationship between the number of sides a polygon has and the number of triangles contained in that polygon? (See the 2<sup>nd</sup> and 4<sup>th</sup> columns above.)

The number of triangles is always 2 less than the number of sides.

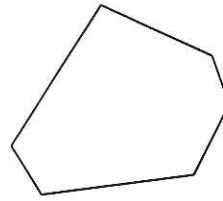
3. Develop a formula for the sum of the interior angles of a polygon if you know the number of sides. Let the number of sides be  $n$  (ex: for an octagon  $n = 8$ ).

$$(n-2) \cdot 180^\circ = \text{The sum of the interior angles of a polygon.}$$

Part 2 - Investigation



Regular Polygon



Nonregular Polygon

1. Compare the two polygons shown above. How would you define a regular polygon versus a nonregular polygon?

Regular: all sides and angles are  $\cong$ .

Nonregular: Sides and angles are  $\neq$ . (Not  $\cong$ )

2. What is the sum of the interior angles of a **hexagon**? See your table on the previous page or use your formula.

$$720^\circ$$

3. What is the measure of **one** angle of a regular hexagon? (See your definition above.)

$$720/6 = 120^\circ \quad (\text{since all } \angle\text{'s are } \cong)$$

4. Knowing the sum of the angles of a regular polygon, how can you find the measure of one angle?

Divide the sum by the number of angles  
(or the number of sides.)

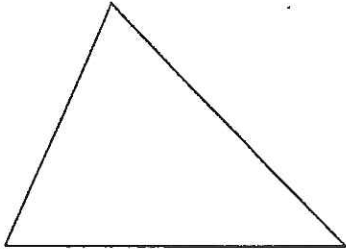
5. Use the information you discovered to complete the table below:

Regular Polygon	Interior angle sum	Measure of one angle
Triangle	$180^\circ \div 3 =$	$60^\circ$
Quadrilateral	$360^\circ \div 4 =$	$90^\circ$
Pentagon	$540^\circ \div 5 =$	$108^\circ$
Hexagon	$720^\circ \div 6 =$	$120^\circ$
Heptagon	$900^\circ \div 7 =$	$128.6^\circ$
Octagon	$1080^\circ \div 8 =$	$135^\circ$
Nonagon	$1260^\circ \div 9 =$	$140^\circ$
Decagon	$1440^\circ \div 10 =$	$144^\circ$
Dodecagon	$1800^\circ \div 12 =$	$150^\circ$
:		
16-gon	$2520^\circ \div 16 =$	$157.5^\circ$

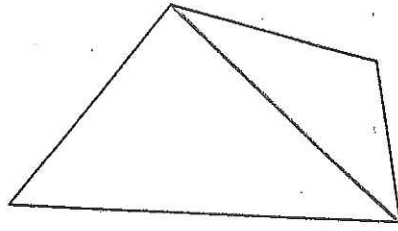
6. Using your formula from the previous page and your chart to the left, find the formula for each angle of a regular polygon, given the number of sides,  $n$ .

$$\frac{(n-2) \cdot 180^\circ}{n} = \text{Each Interior Angle (Regular Polygons only!)}$$

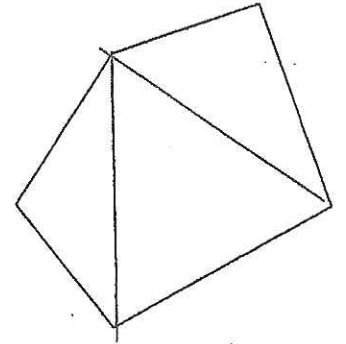
## Interior Angle Sum of Polygons



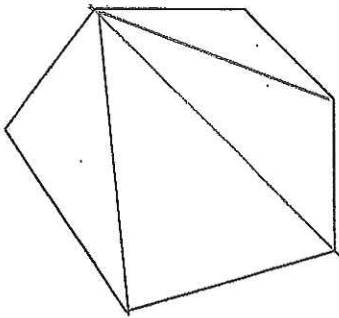
$n = 3$



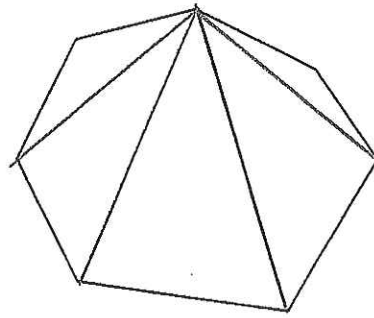
$n = 4$



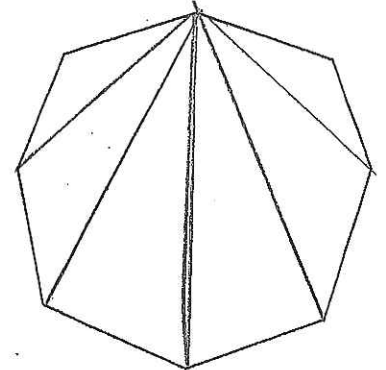
$n = 5$



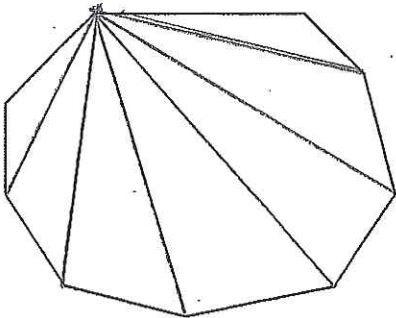
$n = 6$



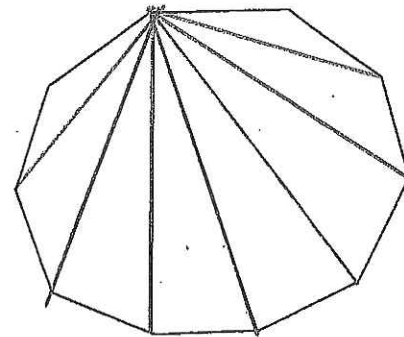
$n = 7$



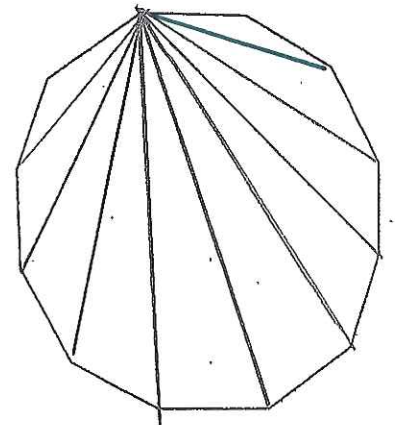
$n = 8$



$n = 9$



$n = 10$



$n = 12$