Math Analysis AB Unit 1 - Day 7

Objectives: Evaluate combinations & compositions of functions.
State the domain of combinations & compositions

TS: Explicitly assess information & draw conclusions

## Combinations of Functions

Let $f$ and $g$ be two functions with overlapping domains. Then, for all $x$ common to both domains, the sum, difference, product and quotient of $f$ and $g$ are:

1.) Sum: $(f+g)(x)=f(x)+g(x)$

2.) Difference: $(f-g)(x)=f(x)-g(x)$

3.) Product: $(fg)(x)=f(x)\cdot g(x)$

4.) Quotient: $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)},\quad g(x)\neq 0$

Example 1: Evaluate each combination of *f* and *g*. State the domain of the combination. (scroll down)

$f(x) = 2x + 1$ $\qquad g(x) = x^2 - 4$

a.) $(f + g)(x) = f(x) + g(x)$

$(2x+1) + (x^2-4)$

$\boxed{x^2+2x-3}$

DOM: $(-\infty, \infty)$

b.) $(f - g)(x) = f(x) - g(x)$

$(2x+1) - (x^2-4)$

$\boxed{-x^2+2x+5}$

DOM: $(-\infty, \infty)$

c.) $(fg)(x) = f(x) \cdot g(x)$

$(2x+1)(x^2-4)$

$2x^3 + x^2 - 8x - 4$

DOM: $(-\infty, \infty)$

d.) $\left(\dfrac{f}{g}\right)(x)$

$\dfrac{2x+1}{x^2-4}$ $\qquad \{x : x \neq \pm 2\}$

DOM: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

e.) $(fg)(3) =$

2 Methods:

① $2(3)^3 + (3)^2 - 8(3) - 4 = 35$

② $f(3) \cdot g(3)$

$7 \cdot 5 = 35$

f.) $(f - g)(-2) =$

2 Methods:

① $-(-2)^2 + 2(-2) + 5 = -3$

② $f(-2) - g(-2)$

$-3 - 0 = -3$

Example 2: Graphs of combinations using the calculator.

Enter: $Y_1 = 2x + 1$

$Y_2 = x^2 - 4$

$Y_3 = Y_1 Y_2$

Only highlight $Y_3$ and we will see the graph of the product *(fg)(x)*.

Use the value feature to evaluate:

$(fg)(3) =$

$(fg)(-2) =$

$(fg)(0.5) =$

## Compositions of Functions

The composition of the function *f* with *g* is

$$(f \circ g)(x) = f(g(x))$$

The domain of the composition is the set of all *x* in the domain of *g* such that *g(x)* is in the domain of *f*.

<u>Domain:</u> ① domain of function going in?
② domain of final answer?

Example 3: Evaluate each composition and state the domain of the composition.

$f(x) = x^2 - 9 \qquad g(x) = \sqrt{9-x^2}$

a.) $(f \circ g)(x) = f(g(x))$

$= f(\sqrt{9-x^2})$

Dom? $[-3,3]$

$= (\sqrt{9-x^2})^2 - 9$

$= 9 - x^2 - 9$

$\boxed{(f \circ g)(x) = -x^2, \ -3 \le x \le 3}$

b.) $(g \circ f)(x) = g(f(x))$

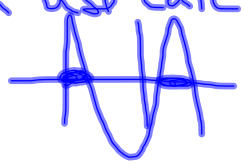
$= g(x^2 - 9)$

$\boxed{(g \circ f)(x) = \sqrt{9-(x^2-9)^2}}$

$9-(x^2-9)^2 \ge 0$ ↙DOM

$\boxed{[-\sqrt{12}, -\sqrt{6}] \cup [\sqrt{6}, \sqrt{12}]}$

OR use calc



$[-3.464, -2.449] \cup [2.449, 3.464]$

Example 4: Evaluate the composition & state the domain of the composition.

$f(x) = \frac{1}{x-2}$ $\qquad$ $g(x) = \sqrt{x}$

a.) $f(g(x)) = \frac{1}{\sqrt{x}-2}$ ; $[0,4) \cup (4,\infty)$

$x \geq 0,\ x \neq 4$

Example 5: Decompose. Find two functions *f* and *g* such that

$$(f \circ g)(x) = h(x)$$

$$h(x) = \frac{4}{(5x+2)^2}$$

$f(x) = \frac{4}{x^2}$

$g(x) = 5x+2$

OR

$f(x) = \frac{4}{(x+2)^2}$

$g(x) = 5x$