Rational Root Theorem
Descartes’ Rule of Signs
Upper & Lower Bound Rule

Unit 5
Warm Up

Is \((x - 3)\) a factor of \(f(x) = 2x^3 - 15x^2 + 22x + 15\)? If so, factor completely.

How many roots will each function have?

a.) \(f(x) = x - 2\)
b.) \(g(x) = 3x^2 + x - 10\)
c.) \(h(x) = 6x^5 + x^3 - 2x^2 + 2\)
Example 1:

How can we find all the zeros of

\[ f(x) = x^4 - x^3 + x^2 - 3x - 6 \]
Rational Root Theorem

If a polynomial $P(x)$ has rational roots then they are of the form $\frac{p}{q}$ where

- $p$ is a factor of the constant term
- $q$ is a factor of the leading coefficient
Example 2: Find all zeros of
\[ f(x) = x^4 - x^3 + x^2 - 3x - 6 \]

\[ p: \pm 1, \pm 2, \pm 3, \pm 6 \]
\[ q: \pm 1 \]

Possible rational roots: \[ \pm 1, \pm 2, \pm 3, \pm 6 \]

\[ f(-1) = 0 \]

\[ \begin{array}{cccc}
-1 & 1 & -1 & 1 & -3 & -6 \\
\hline
 & -1 & 2 & -3 & 6 \\
\hline
 & 1 & -2 & 3 & -6 \\
\end{array} \]

-1 is a lower bound

\[ f(x) = (x+1)(x-2)(x^2+3) \]

Zeros:
\[ x = -1, 2, \pm i\sqrt{3} \]
Refer to example 2:

- What are all the rational roots for ex. 1?
  \[ x = -1, 2 \]

- What are all the real roots for ex. 1?
  \[ x = -1, 2 \]

- What are all the roots for ex. 1?
  \[ x = -1, 2, \pm i\sqrt{3} \]

- Write ex. 1 as a product of linear factors.
  \[ f(x) = (x+1)(x-2)(x+i\sqrt{3})(x-i\sqrt{3}) \]
Example 3: List the possible rational roots of \( f(x) = 2x^3 + 3x^2 - 8x + 3 \)

\[ P : \pm 1, \pm 3 \]

\[ Q : \pm 1, \pm 2 \]

\[ \frac{P}{Q} : \pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2} \]

Possible Rational Roots
Ways to narrow down a long list of rational roots:

- Descartes Rule of Signs
- Upper/Lower Bound Rules
Descartes Rule of Signs

\[ P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \]

1.) # of positive real zeros of \( f \) is equal to the number of sign changes of \( P(x) \) or less than that by an even integer.

2.) # of negative real zeros of \( f \) is equal to the number of sign changes of \( P(-x) \) or less than that by an even integer.
Example 4:

Use Descartes Rule of Signs to determine the # of positive and negative real roots

\[ f(x) = 2x^3 + 3x^2 - 8x + 3 \]

- \( f \) has 2 or 0 positive real roots

\[ f(-x) = -2x^3 + 3x^2 + 8x + 3 \]

- \( f \) has 1 negative real root
Example 5:

How many + and – real roots can
\( f(x) = x^3 - 9x^2 + 27x - 27 \) have?

3 or 1 + real root

\( f(-x) = -x^3 - 9x^2 - 27x - 27 \)

0 – real roots
Upper and Lower Bound Rule

One more test to narrow down the list of roots…

Suppose f(x) is divided by x – c using syn. div.

If c>0 and each number is the last row is either + or 0, c is an upper bound for the real zeros of f. (there is no zero above c)

If c<0 and the numbers in the last row alternate + - (0 can be + or -), c is a lower bound for the real zeros of f. (there is no zero below c)
Example 6:

Find the real zeros.

\[ f(x) = x^4 - 4x^3 + 16x - 16 \]

\[ \frac{p}{q} : \pm 1, \pm 2, \pm 4, \pm 8, \pm 16 \]

\[ f(1) \neq f(-1) \neq 0 \]

\[ f(x) = (x-2)^2(x^2-4) \]

\[ f(x) = (x-2)^3(x+2) \]

\[ \text{Zeros: } x = 2, -2 \]
Homework Assignment (adjusted)

Pg. 124-125 #32, 39-78 (x3)