# 11-8 Probabilities of Disjoint and Overlapping Events

### Essential Skill:
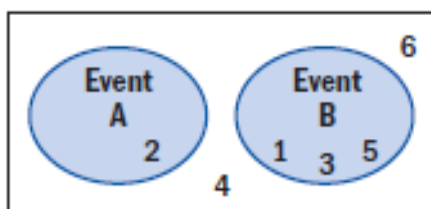Gather and organize information and data.

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Disjoint events, or mutually exclusive events, are events that have no outcomes in common.

Overlapping events are events that have one or more outcomes in common.

**Disjoint Events**

Event A: Roll a 2.

Event B: Roll an odd number.

**Overlapping Events**

Event A: Roll an even number.

Event B: Roll a prime number.

**Example 1**          *Identifying Disjoint and Overlapping Events*

**Tell whether the events are *disjoint* or *overlapping*.**

**a.** Roll a number cube.

   **Event A:** Roll a number less than 4.

   **Event B:** Roll a 5.

**b.** Randomly select a student.

   **Event A:** Select a 7th grader.

   **Event B:** Select a boy.

Event A:          Event B:

1, 2, 3                5

Overlapping!

Disjoint

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Suppose you choose a book to read. Are the events "choosing a hard cover book" and "choosing a fiction book" disjoint or overlapping?

Overlapping

## Probability of Disjoint Events

**Words** For two disjoint events, the probability that either of the events occurs is the sum of the probabilities of the events.

**Algebra** If A and B are disjoint events, then
$P(A \text{ or } B) = P(A) + P(B)$.

**Probability of Disjoint Events** The Venn diagram shows two disjoint events that involve rolling a number cube.

Event A: Roll a number less than 4.

Event B: Roll a number greater than 4.
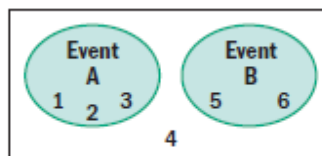
There are 6 possible outcomes. There are 5 favorable outcomes for the event A *or* B. So, $P(A \text{ or } B) = \frac{5}{6}$. You can also find $P(A \text{ or } B)$ by finding the sum of the probability of event A and the probability of event B.

$$P(A \text{ or } B) = P(A) + P(B) = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

This result suggests the following rule.

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**Example 2**    *Finding the Probability of Disjoint Events*

**Raffle** Fifty tickets are sold for a raffle. You buy 2 tickets, and your friend buys 3 tickets. One ticket is randomly chosen as the winning ticket. What is the probability that you or your friend wins the raffle?

$$P\left(\begin{array}{c}\text{You or friend} \\ \text{win} \quad \text{wins}\end{array}\right) = P\left(\begin{array}{c}\text{You} \\ \text{win}\end{array}\right) + P\left(\begin{array}{c}\text{friend} \\ \text{wins}\end{array}\right)$$

$$= \frac{2}{50} + \frac{3}{50}$$

$$= \frac{5}{50}$$

$$= \boxed{\frac{1}{10}}$$

In an election, candidate A received 35% of the vote, candidate B received 22% of the vote, and candidate C received 43% of the vote. If you randomly select a person from all who voted, what is the probability that the person voted for either candidate A *or* candidate B?

$$P(A \text{ or } B) = P(A) + P(B)$$

$$= \frac{35}{100} + \frac{22}{100}$$

$$= \frac{57}{100} = \boxed{57\%}$$

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### Probability of Overlapping Events

**Words**  For two overlapping events, the probability that either of the events occurs is the sum of the probabilities of the events minus the probability of both events.

**Algebra**  If A and B are overlapping events, then
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

**Probability of Overlapping Events**  The Venn diagram shows two overlapping events that involve rolling a number cube.

Event A: Roll a number less than 5.

Event B: Roll an even number.
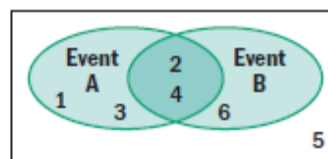
There are 6 possible outcomes. There are 5 favorable outcomes for the event A *or* B. So, $P(A \text{ or } B) = \frac{5}{6}$. There are 2 favorable outcomes for the

event A *and* B. So, $P(A \text{ and } B) = \frac{2}{6}$. These outcomes are counted twice when you find the sum of $P(A)$ and $P(B)$. In order to find $P(A \text{ or } B)$ using the sum of $P(A)$ and $P(B)$, you must subtract $P(A \text{ and } B)$ *once*.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = \frac{4}{6} + \frac{3}{6} - \frac{2}{6} = \frac{5}{6}$$

**Example 3**    *Finding the Probability of Overlapping Events*

You roll two number cubes, one red and one blue. What is the probability that you roll a 4 on at least one of the number cubes?

1,1    2 1    3 1    4 1    5 1   6 1
1,2    2 2    3 2    4 2    5 2   6 2
1 3    2 3    3 3    4 3    5 3   6 3
1 4    2 4    3 4    4 4    5 4   6 4
1 5    2 5    3 5    4 5    5 5   6 5
1 6    2 6    3 6    4 6    5 6   6 6

$$P\left(\begin{smallmatrix}4\\ on\ red\end{smallmatrix}\ or\ \begin{smallmatrix}4\ on\\ blue\end{smallmatrix}\right) = P\left(\begin{smallmatrix}4\ on\\ red\end{smallmatrix}\right) + P\left(\begin{smallmatrix}4\ on\\ Blue\end{smallmatrix}\right) - P\left(\begin{smallmatrix}4\ on\\ red\ \&\ blue\end{smallmatrix}\right)$$

$$= \frac{6}{36} + \frac{6}{36} - \frac{1}{36}$$

$$= \frac{11}{36}$$

## Complementary Events

Two events are **complementary events** if they are disjoint events and one event or the other must occur.

The sum of the probabilities of complementary events is always 1.

**P(not A) = 1 - P(A)**

**Example 4**   *Finding the Probability of Complementary Events*

**Weather** The forecast claims that there is a 40% probability of snow tomorrow. What is the probability that it will *not* snow tomorrow?
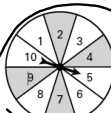
$$P\left(\begin{smallmatrix}\text{will}\\\text{not}\\\text{snow}\end{smallmatrix}\right) = 1 - P\left(\begin{smallmatrix}\text{WILL}\\\text{snow}\end{smallmatrix}\right)$$

$$= 1 - .40$$

$$= .6 \quad \text{or} \quad \boxed{60\%}$$

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**You try:**

1.) The spinner is divided into equal parts. For the specified events A and B, tell whether the events are *disjoint* or *overlapping*. Then find *P(A or B)*.

Event A: Stops on an even number.
Event B: Stops on a shaded sector.

Event A: Stops on a shaded sector.
Event B: Stops on a multiple of 5.

✗ Disjoint

$$P(A \text{ or } B) = P(A) + P(B)$$
$$= \frac{4}{10} + \frac{2}{10}$$
$$= \frac{6}{10} = \boxed{\frac{3}{5}}$$

Overlapping
$$P(A \text{ or } B) = P(A) + P(B) - P(A \& B)$$
$$= \frac{5}{10} + \frac{4}{10} - \frac{3}{10} = \boxed{\frac{7}{10}}$$

2.) You roll two number cubes. What is the probability that you roll a number less than 3 on at least one number cube?

| | | | | | |
|---|---|---|---|---|---|
| 1 1 | 2 1 | 3 1 | 2 1 | 5 1 | 6 1 |
| 1 2 | 2 2 | 3 2 | 4 2 | 5 2 | 6 2 |
| 1 3 | 2 3 | 3 3 | 4 3 | 5 3 | 6 3 |
| 1 4 | 2 4 | 3 4 | 4 4 | 5 4 | 6 4 |
| 1 5 | 2 5 | 3 5 | 4 5 | 5 5 | 6 5 |
| 1 6 | 2 6 | 3 6 | 4 6 | 5 6 | 6 6 |

$$P\left(\begin{smallmatrix}\#<\\3\end{smallmatrix}\text{ or }\begin{smallmatrix}\#<\\3\end{smallmatrix}\right) = P\left(\begin{smallmatrix}<3\text{ on}\\\text{1st roll}\end{smallmatrix}\right) + P\left(\begin{smallmatrix}<3\text{ on}\\\text{2nd roll}\end{smallmatrix}\right) - P\left(\begin{smallmatrix}<3\\\text{on}\\\text{Both}\end{smallmatrix}\right)$$

$$= \frac{12}{36} + \frac{12}{36} - \frac{4}{36}$$

$$= \frac{20}{36}$$

$$= \boxed{\frac{5}{9}}$$