4.7 Piecewise Functions

Essential Skill: Demonstrate Understanding of Concepts

Essential Question

How can you describe a function that is represented by more than one equation?

Work with a partner.

a. Does the graph represent $y$ as a function of $x$? Justify your conclusion.

b. What is the value of the function when $x = 0$? How can you tell?

c. Write an equation that represents the values of the function when $x \leq 0$. $y = -\lvert x \rvert$

d. Write an equation that represents the values of the function when $x > 0$. $\sqrt{2}$

e. Combine the results of parts (c) and (d) to write a single description of the function.

$$f(x) = \begin{cases} 
-\lvert x \rvert, & \text{if } x \leq 0 \\
\sqrt{2}, & \text{if } x > 0
\end{cases}$$
Core Concept

Piecewise Function

A **piecewise function** is a function defined by two or more equations. Each “piece” of the function applies to a different part of its domain. An example is shown below:

\[
f(x) = \begin{cases} 
  x - 2, & \text{if } x \leq 0 \\
  2x + 1, & \text{if } x > 0
\end{cases}
\]

- The expression \( x - 2 \) represents the value of \( f \) when \( x \) is less than or equal to 0.
- The expression \( 2x + 1 \) represents the value of \( f \) when \( x \) is greater than 0.

Evaluate the function.

\[
f(x) = \begin{cases} 
  x - 2, & \text{if } x \leq 0 \\
  2x + 1, & \text{if } x > 0
\end{cases}
\]

a. when \( x = -1 \)

The value of \( f \) is \(-3\) when \( x = -1 \).

b. when \( x = 1 \)

The value of \( f \) is \(3\) when \( x = 1 \).

\[
a) \quad f(-1) = -1 - 2 = -3 \\
\]

\[
b) \quad f(1) = 2(1) + 1 = 3
\]
Graph \( y = \begin{cases} 
-x + 2, & \text{if } x \leq 0 \\
2x, & \text{if } x > 0
\end{cases} \).

Describe the domain and range.

**Domain:** all real #’s \( \mathbb{R} \)

**Range:** \( y > 0 \)

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Graph the function. Describe the domain and range.

1. \( y = \begin{cases} 
x + 1, & \text{if } x \leq 0 \\
-x, & \text{if } x > 0
\end{cases} \)

2. \( y = \begin{cases} 
x - 2, & \text{if } x < 0 \\
4x, & \text{if } x \geq 0
\end{cases} \)
Write a piecewise function for the graph.

\[ f(x) = \begin{cases} 
-x - 2, & x \leq 0 \\
2x - 3, & x > 0 
\end{cases} \]

\[ y = \begin{cases} 
-x - 2, & x \leq 0 \\
2x - 3, & x > 0 
\end{cases} \]
Write a piecewise function for the graph.

**step function**: piecewise function defined by a constant value over each part of the domain

The graph consists of a series of line segments.

\[
f(x) = \begin{cases} 
2, & \text{if } 0 \leq x < 2 \\
3, & \text{if } 2 \leq x < 4 \\
4, & \text{if } 4 \leq x < 6 \\
5, & \text{if } 6 \leq x < 8 \\
6, & \text{if } 8 \leq x < 10 \\
7, & \text{if } 10 \leq x < 12 
\end{cases}
\]
You rent a bicycle for 4 days. The bike store charges $20 for the first day and $15 for each additional day. Write and graph a step function that represents the relationship between the number $x$ of days and the total cost $y$ (in dollars) of renting the bicycle.

\[ f(x) = \begin{cases} 
20, & \text{if } 0 < x \leq 1 \\
35, & \text{if } 1 < x \leq 2 \\
50, & \text{if } 2 < x \leq 3 \\
65, & \text{if } 3 < x \leq 4 
\end{cases} \]
Work on 4.7 Extra Practice Worksheet.
10. 

\[ \text{domain: all real numbers; } \]
\[ \text{range: } y \geq 0 \]

11. 

\[ \text{domain: all real numbers } \]
\[ \text{range: } y > 2 \]

12. 

\[ \text{domain: all real numbers; } \]
\[ \text{range: all real numbers} \]

13. 

\[ \text{domain: all real numbers; } \]
\[ \text{range: } \{-1, 0, 1\} \]

14. 

\[ y = \begin{cases} 
-\frac{7}{3}x, & x < 0 \\
3, & x \geq 0 
\end{cases} \]

15. 

\[ y = \begin{cases} 
3, & -3 \leq x < 0 \\
4, & 0 \leq x \leq 1 \\
x, & 1 < x \leq 5 
\end{cases} \]
16. \[ y = \begin{cases} 4 & 0 < x < 1 \\ 5 & 1 \leq x < 2 \\ 6 & 2 \leq x < 3 \\ 7 & 3 \leq x < 4 \\ 8 & 4 \leq x < 5 \end{cases} \]