PHYSICS C: MECHANICS
SECTION I
Time—45 minutes
35 Questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and then fill in the corresponding oval on the answer sheet.

Note: To simplify calculations, you may use $g = 10\,\text{m/s}^2$ in all problems.

Questions 1-2

Starting from rest, a vehicle accelerates on a straight level road at the rate of 4.0 m/s^2 for 5.0 s.

1. What is the speed of the vehicle at the end of this time interval?
   (A) 1.3 m/s
   (B) 10 m/s
   (C) 20 m/s
   (D) 80 m/s
   (E) 100 m/s

$v_f = v_i + at = 4(5) = 20$ m/s

2. What is the total distance the vehicle travels during this time interval?
   (A) 10 m
   (B) 20 m
   (C) 25 m
   (D) 40 m
   (E) 50 m

$\Delta x = v_i t + \frac{1}{2}at^2 = \frac{1}{2}(4)(5)^2 = 50$ m

3. All of the following are units of power EXCEPT
   (A) watts
   (B) joules per second
   (C) electron volts per second
   (D) newton meters per second
   (E) kilogram meters per second

power = $\frac{\text{Energy}}{\text{Time}}$

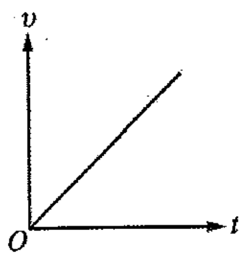
4. A dart gun is used to fire two rubber darts with different but unknown masses, M_1 and M_2. The gun exerts the same constant force on each dart, but its magnitude F is unknown. The magnitudes of the accelerations of both darts, a_1 and a_2, respectively, are measured. Which of the following can be determined from these data?
   (A) F only
   (B) M_1 and M_2 only
   (C) The ratio of M_1 and M_2 only
   (D) F and the ratio of M_1 and M_2 only
   (E) F, M_1, and M_2

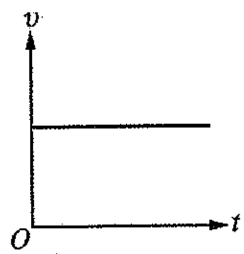
$M_1 = ?$ $M_2 = ?$ $F = ?$

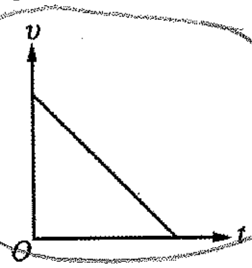
$F = M_1 a_1$ $F = M_2 a_2$

$M_1 a_1 = M_2 a_2$

5. An object is thrown vertically upward in a region where g is constant and air resistance is negligible. Its speed is recorded from the moment it leaves the thrower's hand until it reaches its maximum height. Which of the following graphs best represents the object's speed v versus time t ?

(A)


(B)


(C)


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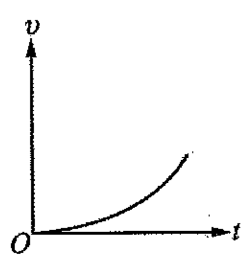
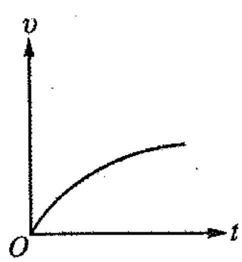


(E)


6. If air resistance is negligible, the speed of a 2 kg sphere that falls from rest through a vertical displacement of 0.2 m is most nearly

(A) 1 m/s
(B) 2 m/s
(C) 3 m/s
(D) 4 m/s
(E) 5 m/s

7. A person holds a portable fire extinguisher that ejects 1.0 kg of water per second horizontally at a speed of 6.0 m/s. What horizontal force in newtons must the person exert on the extinguisher in order to prevent it from accelerating?

(A) 0 N
(B) 6 N
(C) 10 N
(D) 18 N
(E) 36 N

8. A disk is free to rotate about an axis perpendicular to the disk through its center. If the disk starts from rest and accelerates uniformly at the rate of 3 radians/s^2 for 4 s, its angular displacement during this time is

(A) 6 radians
(B) 12 radians
(C) 18 radians
(D) 24 radians
(E) 48 radians

Questions 9-10

A 2 kg mass connected to a spring oscillates on a horizontal, frictionless surface with simple harmonic motion of amplitude 0.4 m. The spring constant is 50 N/m.

9. The period of this motion is

(A) 0.04π s
(B) 0.08π s
(C) 0.4π s
(D) 0.8π s
(E) 1.26π s

$\omega = \sqrt{\frac{k}{m}} = 2\pi\left(\frac{1}{T}\right)$

$T = \sqrt{\frac{m}{k}}\, 2\pi = \sqrt{\frac{2}{50}}\, 2\pi = \frac{2}{5}\pi = .4\pi$

10. The maximum velocity occurs where the

(A) potential energy is a maximum
(B) kinetic energy is a minimum
(C) displacement from equilibrium is equal to the amplitude of 0.4 meter
(D) displacement from equilibrium is half the amplitude
(E) displacement from equilibrium is equal to zero

11. A student is asked to determine the mass of Jupiter. Knowing which of the following about Jupiter and one of its moons will allow the determination to be made?

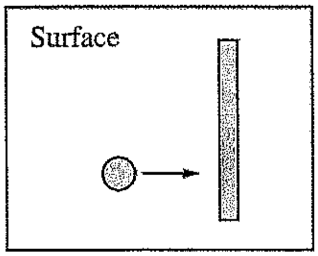
I. The time it takes for Jupiter to orbit the Sun
II. The time it takes for the moon to orbit Jupiter
III. The average distance between the moon and Jupiter

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

$\frac{GM_J m}{r^2} = m\frac{v^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2$

$\frac{GM_J m}{r^2} = \frac{4\pi^2 r m}{T^2}$

$M_J = \frac{4\pi^2}{G} \cdot \frac{r^3}{T^2}$

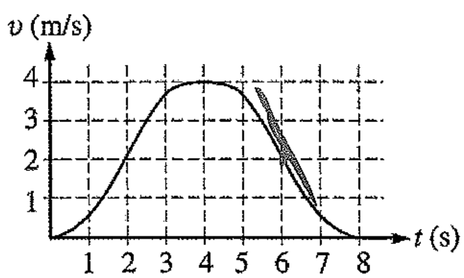


Top View

12. A disk sliding on a horizontal surface that has negligible friction collides with a rod that is free to move and rotate on the surface, as shown in the top view above. Which of the following quantities must be the same for the disk-rod system before and after the collision?

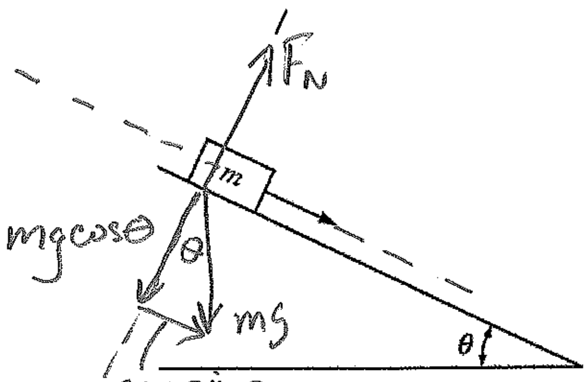
    I. Linear momentum
    II. Angular momentum
    III. Kinetic energy

    (A) I only
    (B) II only
    (C) I and II only
    (D) II and III only
    (E) I, II, and III



13. The velocity v of an elevator moving upward between adjacent floors is shown as a function of time t in the graph above. At which of the following times is the force exerted by the elevator floor on a passenger the least?

    (A) 1 s
    (B) 3 s
    (C) 4 s
    (D) 5 s
    (E) 6 s



14. An object of mass m moves with acceleration a down a frictionless incline that makes an angle with the horizontal, as shown above. If N is the normal force exerted by the plane on the block, which of the following is correct?

    (A) $N = mg$
    (B) $N = ma$
    (C) $a = mg\sin\theta$
    (D) $a = g\sin\theta$
    (E) $a = mg\cos\theta$

15. A disc of mass m slides with negligible friction along a flat surface with a velocity v. The disc strikes a wall head-on and bounces back in the opposite direction with a kinetic energy one-fourth of its initial kinetic energy. What is the final velocity of the disc?

    (A) $v/4$
    (B) $v/2$
    (C) $-v$
    (D) $-v/2$
    (E) $-v/4$

Questions 16-18

The following pairs of equations show how the x- and y-coordinates of a particle vary with time t. In the equations, A, B, and ω are nonzero constants. Choose the pair of equations that best answers each of the following questions. A choice may be used once, more than once, or not at all.

(A) $x = A\cos\omega t$
$y = A\sin\omega t$

(B) $x = A\cos\omega t$
$y = 2A\cos\omega t$

(C) $x = At$
$y = Bt$

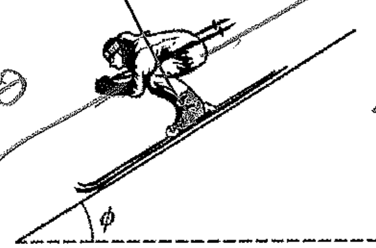
(D) $x = At^2$
$y = Bt^2$

(E) $x = At$
$y = Bt^2$

16. Which pair of equations can describe the path of a particle moving with zero acceleration?

17. Which pair of equations can describe the path of a particle moving with an acceleration that is perpendicular to the velocity of the particle at $t = 0$ and remains constant in magnitude and direction?

18. Which pair of equations can describe the path of a particle that moves with a constant speed and with a nonzero acceleration that is constant in magnitude?



19. A skier slides at constant speed down a slope inclined at an angle ϕ to the horizontal, as shown above. If air resistance is negligible, the coefficient of friction μ between the skis and the snow is equal to

(A) $\dfrac{1}{\tan\phi}$

(B) $\dfrac{1}{\cos\phi}$

(C) $\sin\phi$

(D) $\cos\phi$

(E) $\tan\phi$

20. A 2000 kg car, initially at rest, is accelerated along a horizontal roadway at 3 m/s^2. What is the average power required to bring the car to a final speed of 20 m/s?

(A) 6×10^3 W
(B) 2×10^4 W
(C) 3×10^4 W
(D) 4×10^4 W
(E) 6×10^4 W

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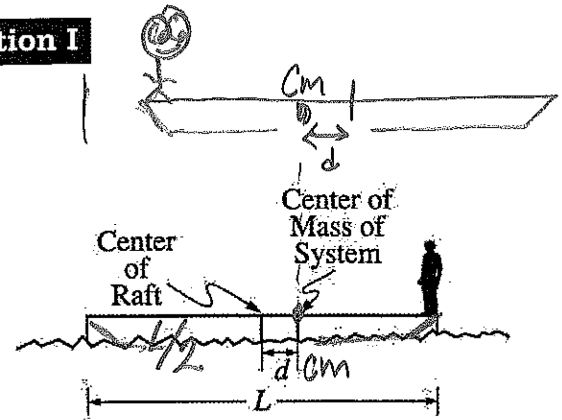


21. A person is standing at one end of a uniform raft of length L that is floating motionless on water, as shown above. The center of mass of the person-raft system is a distance d from the center of the raft. The person then walks to the other end of the raft. If friction between the raft and the water is negligible, how far does the raft move relative to the water?

(A) $\frac{L}{2}$

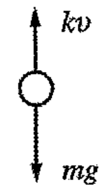
(B) L

(C) $\frac{d}{2}$

(D) d

(E) $2d$

CM of system does not move
∴ boat moves 2d



22. The object of mass m shown above is dropped from rest near Earth's surface and experiences a resistive force of magnitude kv, where v is the speed of the object and k is a constant. Which of the following expressions can be used to find v as a function of time t ? (Assume that the direction of the gravitational force is positive.)

(A) $\int_0^v \frac{dv}{mg - kv} = \int_0^t \frac{dt}{m}$

(B) $\int_0^t \frac{dv}{mg - kv} = \int_0^v \frac{dt}{m}$

(C) $\int_0^v \frac{dv}{kv} = \int_0^t \frac{dt}{m}$

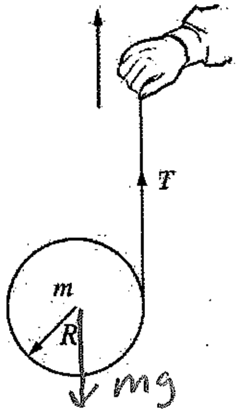
(D) $\int_0^v (mg - kv)\, dv = \int_0^t m\, dt$

(E) $\int_0^v (mg - kv)\, dt = \int_0^t m\, dv$

$kv - mg = -m\frac{dv}{dt}$

$mg - kv = m\frac{dv}{dt}$

Questions 23-24



A solid cylinder of mass m and radius R has a string wound around it. A person holding the string pulls it vertically upward, as shown above, such that the cylinder is suspended in midair for a brief time interval Δt and its center of mass does not move. The tension in the string is T, and the rotational inertia of the cylinder about its axis is $\frac{1}{2}mR^2$.

23. The net force on the cylinder during the time interval Δt is

(A) T
(B) mg
(C) $T - mgR$
(D) $mgR - T$
(E) zero

Suspended means $T - mg = 0$

24. The linear acceleration of the person's hand during the time interval Δt is

(A) $\frac{T - mg}{m}$
(B) $2g$
(C) $\frac{g}{2}$
(D) $\frac{T}{m}$
(E) zero

$TR = I\alpha$

$TR = \frac{1}{2}mR^2\left(\frac{a_{tan}}{R}\right)$

$a_{tan} = \frac{2T}{m}$

w/ $T = mg$

$a_{tan} = \frac{2(mg)}{m} = 2g$

25. A figure skater goes into a spin with arms fully extended. Which of the following describes the changes in the rotational kinetic energy and angular momentum of the skater as the skater's arms are brought toward the body?

	Rotational Kinetic Energy	Angular Momentum
(A)	Remains the same	Increases
(B)	Remains the same	Remains the same
(C)	Increases	Remains the same
(D)	Decreases	Increases
(E)	Decreases	Remains the same

26. Objects 1 and 2 have the same momentum. Object 1 can have more kinetic energy than object 2 if, compared with object 2, it

(A) has more mass
(B) has the same mass
(C) is moving at the same speed
(D) is moving slower
(E) is moving faster

$m_1 v_1 = m_2 v_2$

Since $KE = \frac{1}{2}mv^2$

27. A 5 kg object is propelled from rest at time $t = 0$ by a net force $\mathbf{F}$ that always acts in the same direction. The magnitude of $\mathbf{F}$ in newtons is given as a function of t in seconds by $F = 0.5t$. What is the speed of the object at $t = 4$ s?

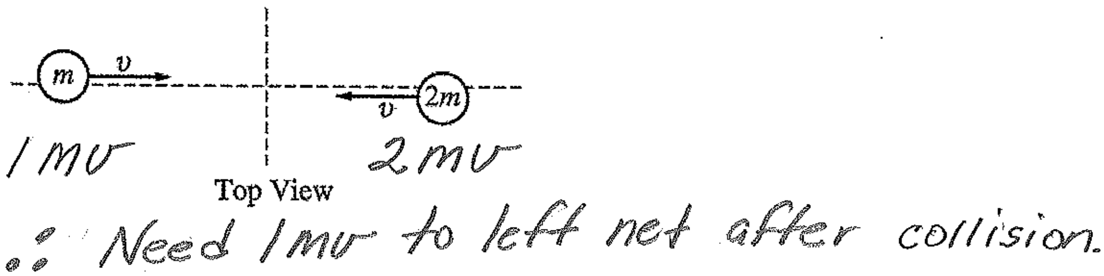
(A) 0.5 m/s
(B) 0.8 m/s
(C) 2.0 m/s
(D) 4.0 m/s
(E) 8.0 m/s

$\Delta v = \int_0^4 \frac{F}{m}\,dt$

$v_f - v_i^{\,0} = \int_0^4 \frac{.5}{5}\,t\,dt$

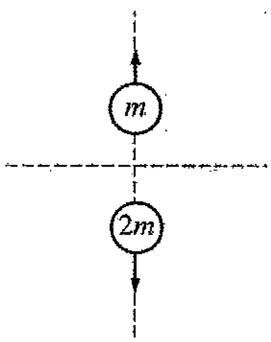
$v_f = \frac{1}{10}\frac{t^2}{2}\Big|_0^4$

$v_f = \frac{8}{10} = .8$ m/s

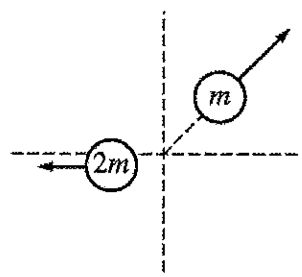


28. Two balls with masses m and $2m$ approach each other with equal speeds v on a horizontal frictionless table, as shown in the top view above. Which of the following shows possible velocities of the balls for a time soon after the balls collide?

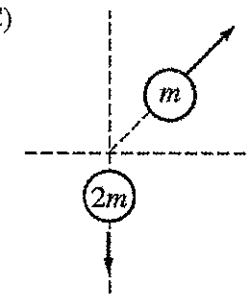
(A)



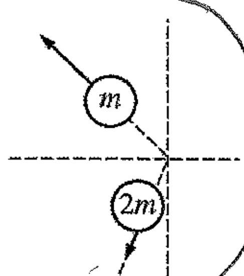
(B)



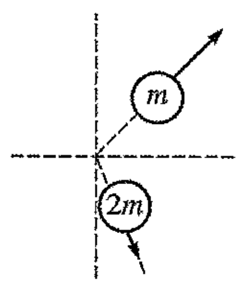
(C)



(D)



(E)



29. A projectile is launched from level ground with an initial speed v_0 at an angle θ with the horizontal. If air resistance is negligible, how long will the projectile remain in the air?

(A) $2v_0/g$
(B) $2v_0 \cos\theta/g$
(C) $v_0 \cos\theta/g$
(D) $v_0 \sin\theta/g$
(E) $2v_0 \sin\theta/g$

$\Delta y = v_0 \sin\theta\, t - \frac{1}{2} g t^2$

$0 = t\,(v_0 \sin\theta - \frac{1}{2} g t)$

$t = 2 v_0 \sin\theta / g$

30. One end of a string is fixed. An object attached to the other end moves on a horizontal plane with uniform circular motion of radius R and frequency f. The tension in the string is F_s. If both the radius and frequency are doubled, the tension is

(A) $\frac{1}{4} F_s$
(B) $\frac{1}{2} F_s$
(C) $2F_s$
(D) $4F_s$
(E) $8F_s$

$F_s = m\frac{v^2}{r}$ $v = \frac{2\pi r}{T}$ $v = 2\pi r f$

$F_s = \frac{m(4\pi^2 r^2 f^2)}{r}$

$F_s = m 4\pi^2 r f^2$

$F_s = 2(4) = 8(m 4\pi^2 r f^2)$

31. An object of unknown mass is initially at rest and dropped from a height h. It reaches the ground with a velocity v_1. The same object is then raised again to the same height h but this time is thrown downward with velocity v_1. It now reaches the ground with a new velocity v_2. How is v_2 related to v_1?

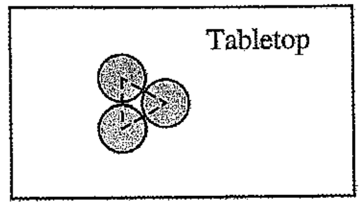
(A) $v_2 = v_1/2$
(B) $v_2 = v_1$
(C) $v_2 = \sqrt{2}\, v_1$
(D) $v_2 = 2v_1$
(E) $v_2 = 4v_1$

$mgh = \frac{1}{2} m v_1^2$

$mgh + \frac{1}{2} m v_1^2 = \frac{1}{2} m v_2^2$

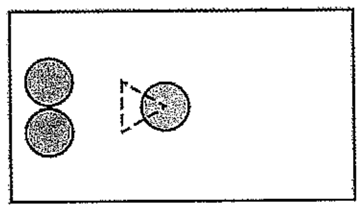
$2(\frac{1}{2} m v_1^2) = \frac{1}{2} m v_2^2$

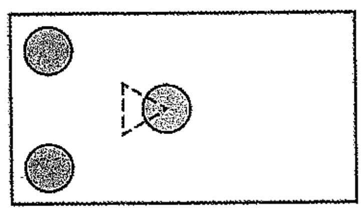
$v_2 = \sqrt{2}\, v_1$

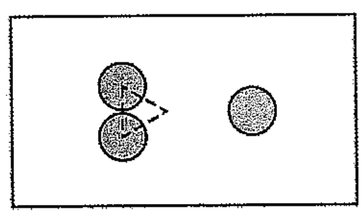


$\vec{p}_i = \vec{p}_f$ (= 0)

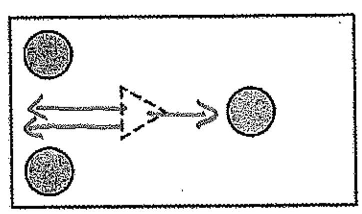
32. Three identical disks are initially at rest on a frictionless, horizontal table with their edges touching to form a triangle, as shown in the top view above. An explosion occurs within the triangle, propelling the disks horizontally along the surface. Which of the following diagrams shows a possible position of the disks at a later time? (In these diagrams, the triangle is shown in its original position.)

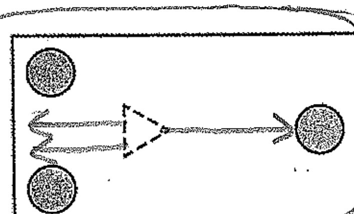
(A)


(B)


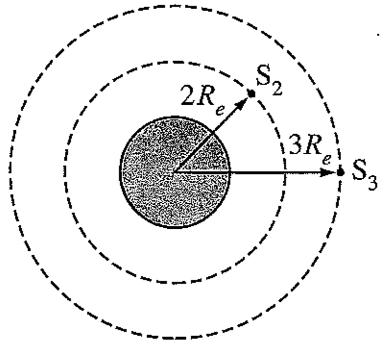
(C)


(A)–(C): $\vec{p}_f \neq 0$

(D)


(E)


The mv to the right has to match the sum 2mv to the left.



33. The figure above represents satellites S_2 and S_3 of equal mass orbiting Earth in circles of radii $2R_e$ and $3R_e$, respectively, where R_e is the radius of Earth. How do the kinetic energy and the angular momentum of S_3 compare with those of S_2 ?

	Kinetic Energy	Angular Momentum
(A)	Less for S_3	Greater for S_3
(B)	Greater for S_3	Greater for S_3
(C)	The same for both	The same for both
(D)	Less for S_3	Less for S_3
(E)	Greater for S_3	Less for S_3

$v_3 < v_2$

$\frac{1}{2}mv^2 = \frac{1}{2}\left|\frac{GMm}{r}\right|$

$v = \sqrt{\frac{GM}{r}}$

$mr^2\omega = mr^2\frac{v}{r}$

$L = mrv$

$v \propto \frac{1}{\sqrt{r}}$

$\therefore L \propto \frac{r}{\sqrt{r}} = \frac{r^{2/2}}{r^{1/2}}$

$L \propto r^{1/2}$

$\therefore L_3 > L_2$

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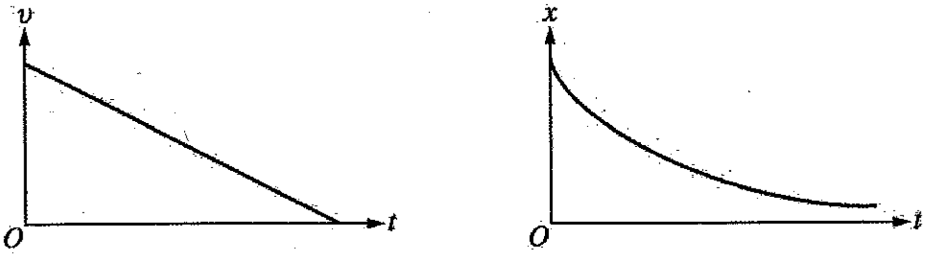
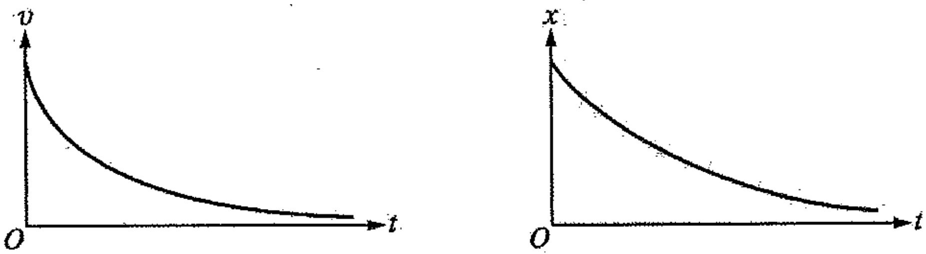
34. A car is traveling along a straight, level road when it runs out of gas at time $t = 0$. From this time on, the net force on the car is a resistive force of $-k\mathbf{v}$, where $\mathbf{v}$ is velocity and k is a constant. Which of the following pairs of graphs best represents the speed v and position x of the car as functions of time after $t = 0$?

(A)



(B)



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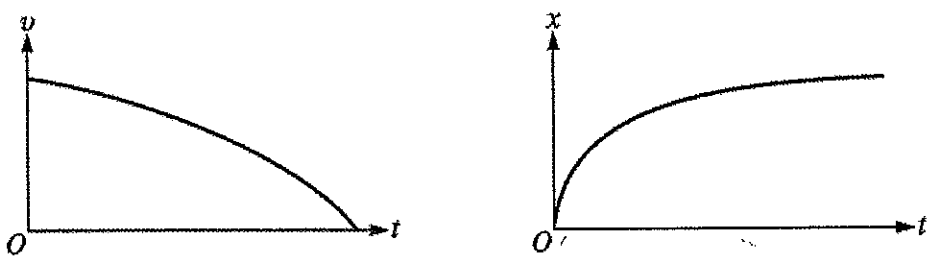
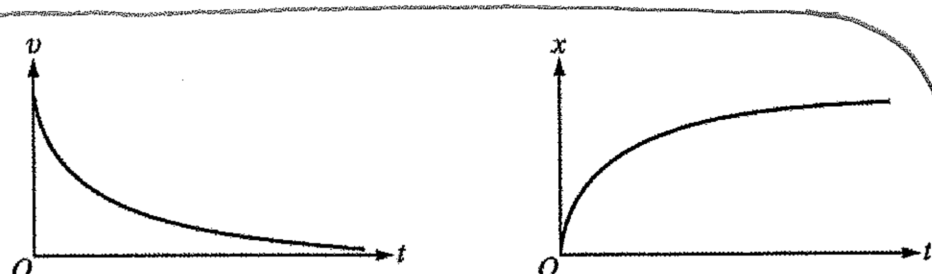
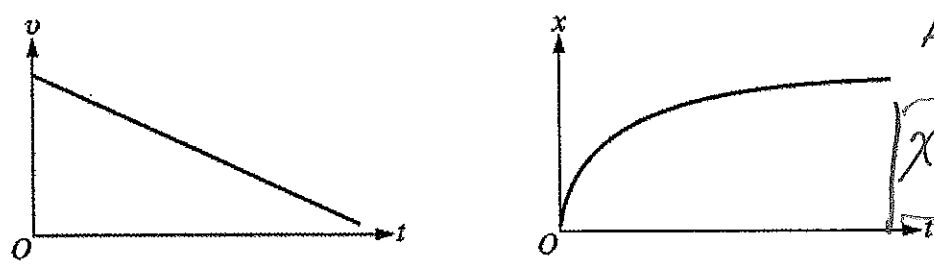


(D)



(E)



$$F = ma$$

$$-kv = m\frac{dv}{dt}$$

$$dt = -\frac{m}{k}\frac{1}{v}\,dv$$

$$t = -\frac{m}{k}\ln v\Big|_{v_0}^{v}$$

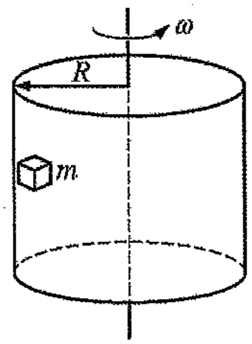
$$-\frac{t}{\tau} = \ln v - \ln v_0 = \ln\frac{v}{v_0}$$

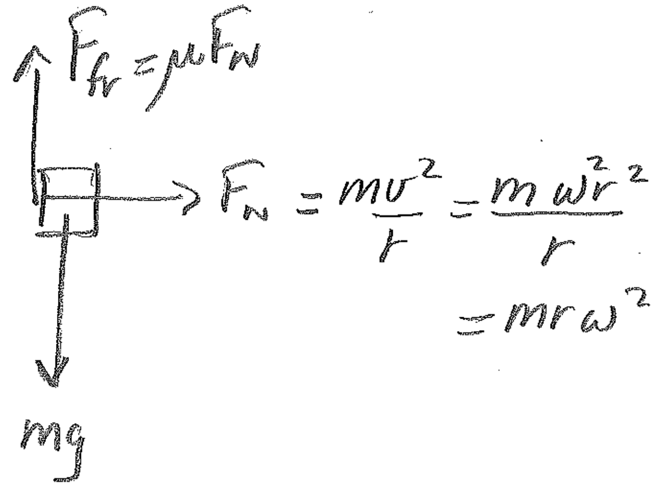
$$v_0 e^{-t/\tau} = v$$

$$\Delta x = \int_0^t v(t)\,dt$$

$$\Delta x = -\tau\int_0^t v_0 e^{-t/\tau}\,\frac{-dt}{\tau}$$

$$x = x_0 - \tau v_0\left(e^{-t/\tau}\right)$$





35. A block of mass m is placed against the inner wall of a hollow cylinder of radius R that rotates about a vertical axis with a constant angular velocity ω, as shown above. In order for friction to prevent the mass from sliding down the wall, the coefficient of static friction μ between the mass and the wall must satisfy which of the following inequalities?

(A) $\mu \geq mg$

(B) $\mu \geq \frac{g}{\omega^2 R}$

(C) $\mu \geq \frac{\omega^2 R}{g}$

(D) $\mu \leq \frac{g}{\omega^2 R}$

(E) $\mu \leq \frac{\omega^2 R}{g}$

$\mu F_N = mg$

$\mu m r \omega^2 = mg$

$\mu = \frac{g}{r\omega^2}$

STOP

END OF MECHANICS SECTION I