Applications of Percents Information & Background (from Wikipedia)

Including fraction and decimal reference pages

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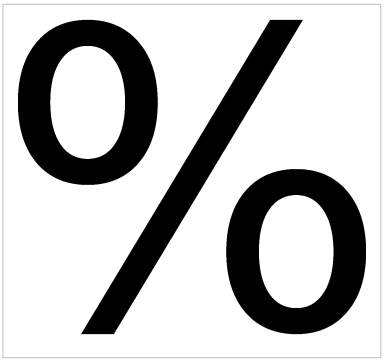
Percentage



In mathematics, a **percentage** is a number or ratio expressed as a fraction of 100. It is often denoted using the percent sign, "%", or the abbreviation "pct."

For example, 45% (read as "forty-five percent") is equal to 45/100, or 0.45. A related system which expresses a number as a fraction of 1,000 uses the terms "per mil" and "millage". Percentages are used to express how large or small one quantity is relative to another quantity. The first quantity usually represents a part of, or a change in, the second quantity. For example, an increase of $ 0.15 on a price of $ 2.50 is an increase by a fraction of 0.15/2.50 = 0.06. Expressed as a percentage, this is therefore a 6% increase. The word 'percent' means 'out of 100' or 'per 100'.

Percentages are usually used to express values between zero and one. However, it is possible to express any ratio as a percentage; for example, 111% is 1.11 and −35% is −0.35.

History

In Ancient Rome, long before the existence of the decimal system, computations were often made in fractions which were multiples of 1/100. For example Augustus levied a tax of 1/100 on goods sold at auction known as *centesima rerum venalium*. Computation with these fractions was similar to computing percentages. As denominations of money grew in the Middle Ages, computations with a denominator of 100 become more standard and from the late 15th century to the early 16th century it became common for arithmetic texts to include such computations. Many of these texts applied these methods to profit and loss, interest rates, and the Rule of Three. By the 17th century it was standard to quote interest rates in hundredths.

Percent sign

The word "percent" is derived from the Latin *per centum* meaning "by the hundred".[1] The percent sign evolved by gradual contraction of the Italian term *per cento*, meaning "for a hundred". The "per" was often abbreviated as "p." and eventually disappeared entirely. The "cento" was contracted to two circles separated by a horizontal line from which the modern "%" is derived.[2]

Calculations

The percent value is computed by multiplying the numeric value of the ratio by 100. For example, to find 50 apples as a percentage of 1250 apples, first compute the ratio 50/1250 = 0.04, and then multiply by 100 to obtain 4%. The percent value can also be found by multiplying first, so in this example the 50 would be multiplied by 100 to give 5,000, and this result would be divided by 1250 to give 4%.

To calculate a percentage of a percentage, convert both percentages to fractions of 100, or to decimals, and multiply them. For example, 50% of 40% is:

$$(50/100) \times (40/100) = 0.50 \times 0.40 = 0.20 = 20/100 = 20\%.$$

It is not correct to divide by 100 and use the percent sign at the same time. (E.g. 25% = 25/100 = 0.25, not 25% / 100, which actually is (25/100) / 100 = 0.0025. A term such as (100/100)% would also be incorrect, this would be read as (1) percent even if the intent was to say 100%.)

Whenever we talk about a percentage, it is important to specify what it is relative to, i.e. what is the total that corresponds to 100%. The following problem illustrates this point.

> In a certain college 60% of all students are female, and 10% of all students are computer science majors. If 5% of female students are computer science majors, what percentage of computer science majors are female?

We are asked to compute the ratio of female computer science majors to all computer science majors. We know that 60% of all students are female, and among these 5% are computer science majors, so we conclude that (60/100) × (5/100) = 3/100 or 3% of all students are female computer science majors. Dividing this by the 10% of all students that are computer science majors, we arrive at the answer: 3%/10% = 30/100 or 30% of all computer science majors are female.

This example is closely related to the concept of conditional probability.

Percentage increase and decrease

Sometimes due to inconsistent usage, it is not always clear from the context what a percentage is relative to. When speaking of a "10% rise" or a "10% fall" in a quantity, the usual interpretation is that this is relative to the *initial value* of that quantity. For example, if an item is initially priced at $200 and the price rises 10% (an increase of $20), the new price will be $220. Note that this final price is 110% of the initial price (100% + 10% = 110%).

Some other examples of percent changes:

- An increase of 100% in a quantity means that the final amount is 200% of the initial amount (100% of initial + 100% of increase = 200% of initial); in other words, the quantity has doubled.
- An increase of 800% means the final amount is 9 times the original (100% + 800% = 900% = 9 times as large).
- A decrease of 60% means the final amount is 40% of the original (100% − 60% = 40%).
- A decrease of 100% means the final amount is *zero* (100% − 100% = 0%).

In general, a change of x percent in a quantity results in a final amount that is $100 + x$ percent of the original amount (equivalently, $1 + 0.01x$ times the original amount).

Compounding percentages

It is important to understand that percent changes, as they have been discussed here, *do not add* in the usual way, if applied sequentially. For example, if the 10% increase in price considered earlier (on the $200 item, raising its price to $220) is followed by a 10% decrease in the price (a decrease of $22), the final price will be $198, *not* the original price of $200. The reason for the apparent discrepancy is that the two percent changes (+10% and −10%) are measured relative to *different* quantities ($200 and $220, respectively), and thus do not "cancel out".

In general, if an increase of x percent is followed by a decrease of x percent, and the initial amount was p, the final amount is $p((1 + 0.01x)(1 - 0.01x)) = p(1 - (0.01x)^2)$; thus the net change is an overall decrease by x percent *of* x percent (the square of the original percent change when expressed as a decimal number). Thus, in the above example, after an increase and decrease of $x = 10$ percent, the final amount, $198, was 10% of 10%, or 1%, less than the initial amount of $200.

This can be expanded for a case where you do not have the same percent change. If the initial percent change is x and the second percent change is y, and the initial amount was p, then the final amount is $p((1 + 0.01x)(1 + 0.01y))$. To change the above example, after an increase of $x = 10$ and decrease of $y = -5$ percent, the final amount, $209, is 4.5% more than the initial amount of $200.

In the case of interest rates, it is a common practice to state the percent change differently. If an interest rate rises from 10% to 15%, for example, it is typical to say, "The interest rate increased by 5%" — rather than by 50%, which would be correct when measured as a percentage of the initial rate (i.e., from 0.10 to 0.15 is an increase of 50%). Such ambiguity can be avoided by using the term "percentage points". In the previous example, the interest rate "increased by 5 percentage points" from 10% to 15%. If the rate then drops by 5 percentage points, it will return to the initial rate of 10%, as expected.

Word and symbol

In British English, *percent* is sometimes written as two words (*per cent*, although *percentage* and *percentile* are written as one word). In American English, *percent* is the most common variant (but cf. *per mille* written as two words).

In the early part of the twentieth century, there was a dotted abbreviation form *"per cent.",* as opposed to *"per cent"*. The form "per cent." is still in use as a part of the highly formal language found in certain documents like commercial loan agreements (particularly those subject to, or inspired by, common law), as well as in the Hansard transcripts of British Parliamentary proceedings. While the term has been attributed to Latin *per centum*, this is a pseudo-Latin construction and the term was likely originally adopted from the French *pour cent*.[*citation needed*] The concept of considering values as parts of a hundred is originally Greek. The symbol for percent (%) evolved from a symbol abbreviating the Italian *per cento*. In some other languages, the form *prosent* is used instead. Some languages use both a word derived from *percent* and an expression in that language meaning the same thing, e.g. Romanian *procent* and *la sută* (thus, *10 %* can be read or sometimes written *ten for [each] hundred*, similarly with the English *one out of ten*). Other abbreviations are rarer, but sometimes seen.

Grammar and style guides often differ as to how percentages are to be written. For instance, it is commonly suggested that the word percent (or per cent) be spelled out in all texts, as in "1 percent" and not "1%". Other guides prefer the word to be written out in humanistic texts, but the symbol to be used in scientific texts. Most guides agree that they always be written with a numeral, as in "5 percent" and not "five percent", the only exception being at the beginning of a sentence: "Ten percent of all writers love style guides." Decimals are also to be used instead of fractions, as in "3.5 percent of the gain" and not "3 ½ percent of the gain". It is also widely accepted to use the percent symbol (%) in tabular and graphic material.

In line with common English practice, style guides—such as *The Chicago Manual of Style*—generally state that the number and percent sign are written without any space in between. However, the International System of Units and the ISO 31-0 standard require a space.

Related units

- Percentage point
- Per mille (‰) 1 part in 1,000
- Basis point (‱) 1 part in 10,000
- Per cent mille (pcm) 1 part in 100,000
- Parts-per notation
- Concentration
- Grade (slope)
- Per-unit system

Other uses

The word "percentage" is often a misnomer in the context of sports statistics, when the referenced number is expressed as a decimal proportion, not a percentage: "The Phoenix Suns' Shaquille O'Neal led the NBA with a .609 field goal percentage (FG%) during the 2008-09 season." (O'Neal made 60.9% of his shots, not 0.609%.) Likewise, the winning percentage of a team, the fraction of matches that the club has won, is also usually expressed as a decimal proportion; a team that has a .500 winning percentage has won 50% of their matches. The practice is probably related to the similar way that batting averages are quoted.

As "percent" it is used to describe the steepness of the slope of a road or railway, formula for which is $100\frac{\text{rise}}{\text{run}}$ which could also be expressed as the tangent of the angle of inclination times 100. The is the ratio of distances a

vehicle would advance vertically and horizontally, respectively, when going up- or downhill, expressed in percent.

Percentage is also used to express composition of a mixture by mass percent and mole percent.

Practical applications

- Baker percentage
- Volume percent

References

[1] American Heritage Dictionary of the English Language, 3rd ed. (1992) Houghton Mifflin
[2] Smith p. 250

External links

- Math "Describing the Meaning of Percent" (http://www.khanacademy.org/video/describing-the-meaning-of-percent?playlist=Developmental) Khan Academy module (first of seven)
- Percentage Calculator (http://percentagecalculator.info) - an educational tool that helps kids grasp the idea of percentages. It calculates percentages and solves a few related problems on the fly, showing how different formulas relate to each other.

Relative change and difference

In any quantitative science, the terms **relative change** and **relative difference** are used to compare two quantities while taking into account the "sizes" of the things being compared. The comparison is expressed as a ratio and is a unitless number. By multiplying these ratios by 100 they can be expressed as percentages so the terms **percentage change**, **percent(age) difference**, or **relative percentage difference** are also commonly used. The distinction between "change" and "difference" depends on whether or not one of the quantities being compared is considered a *standard* or *reference* or *starting* value. When this occurs, the term *relative change* (with respect to the reference value) is used and otherwise the term *relative difference* is preferred. Relative difference is often used as a quantitative indicator of quality assurance and quality control for repeated measurements where the outcomes are expected to be the same. A special case of percent change (relative change expressed as a percentage) called percent error occurs in measuring situations where the reference value is the accepted or actual value (perhaps theoretically determined) and the value being compared to it is experimentally determined (by measurement).

Definitions

Given two numerical quantities, x and y, their *difference*, $\Delta = x - y$, can be called their *actual difference*. When y is a *reference value* (a theoretical/actual/correct/accepted/optimal/starting, etc. value; the value that x is being compared to) then Δ is called their *actual change*. When there is no reference value, the sign of Δ has little meaning in the comparison of the two values since it doesn't matter which of the two values is written first, so one often works with $|\Delta| = |x - y|$, the absolute difference instead of Δ, in these situations. Even when there is a reference value, if it doesn't matter whether the compared value is larger or smaller than the reference value, the absolute difference can be considered in place of the actual change.

The absolute difference between two values is not always a good way to compare the numbers. For instance, the absolute difference of 1 between 6 and 5 is more significant than the same absolute difference between 100,000,001 and 100,000,000. We can adjust the comparison to take into account the "size" of the quantities involved, by defining, for positive values of $x_{reference}$:

$$\text{Relative change}(x, x_{reference}) = \frac{\text{Actual change}}{x_{reference}} = \frac{\Delta}{x_{reference}} = \frac{x - x_{reference}}{x_{reference}}.$$

The relative change is not defined if the reference value ($x_{reference}$) is zero.

For values greater than the reference value, the relative change should be a positive number and for values that are smaller, the relative change should be negative. The formula given above behaves in this way only if $x_{reference}$ is positive, and reverses this behavior if $x_{reference}$ is negative. For example, if we are calibrating a thermometer which reads -6° C when it should read -10° C, this formula for relative change (which would be called *relative error* in this application) gives ((-6) - (-10)) / (-10) = 4/-10 = -0.4, yet the reading is too high. To fix this problem we alter the definition of relative change so that it works correctly for all nonzero values of $x_{reference}$:

$$\text{Relative change}(x, x_{reference}) = \frac{\text{Actual change}}{|x_{reference}|} = \frac{\Delta}{|x_{reference}|} = \frac{x - x_{reference}}{|x_{reference}|}.$$

If the relationship of the value with respect to the reference value (that is, larger or smaller) does not matter in a particular application, the absolute difference may be used in place of the actual change in the above formula to produce a value for the relative change which is always non-negative.

Defining relative difference is not as easy as defining relative change since there is no "correct" value to scale the absolute difference with. As a result, there are many options for how to define relative difference and which one is used depends on what the comparison is being used for. In general we can say that the absolute difference |Δ| is being scaled by some function of the values *x* and *y*, say *f*(*x*,*y*).

$$\text{Relative difference}(x, y) = \frac{\text{Absolute difference}}{|f(x, y)|} = \frac{|\Delta|}{|f(x, y)|} = \left|\frac{x - y}{f(x, y)}\right|.$$

As with relative change, the relative difference is undefined if *f*(*x*,*y*) is zero.

Several common choices for the function *f*(*x*, *y*) would be:

- max (|*x*|,|*y*|),
- max (*x*, *y*),
- min (|*x*|, |*y*|),
- min (*x*, *y*),
- (*x* + *y*)/2, and
- (|*x*| + |*y*|)/2.

Formulas

Measures of relative difference are unitless numbers expressed as a fraction. Corresponding values of percent difference would be obtained by multiplying these values by 100.

One way to define the relative difference of two numbers is to take their absolute difference divided by the maximum absolute value of the two numbers.

$$d_r = \frac{|x - y|}{\max(|x|, |y|)}$$

if at least one of the values does not equal zero. This approach is especially useful when comparing floating point values in programming languages for equality with a certain tolerance.[1] Another application is in the computation of approximation errors when the relative error of a measurement is required.

Another way to define the relative difference of two numbers is to take their absolute difference divided by some functional value of the two numbers, for example, the absolute value of their arithmetic mean:

$$d_r = \frac{|x - y|}{\left(\frac{|x+y|}{2}\right)}.$$

This approach is often used when the two numbers reflect a change in some single underlying entity.[*citation needed*] A problem with the above approach arises when the functional value is zero. In this example, if x and y have the same magnitude but opposite sign, then

$$\frac{|x+y|}{2} = 0,$$

which causes division by 0. So it may be better to replace the denominator with the average of the absolute values of *x* and *y*:[*citation needed*]

$$d_r = \frac{|x-y|}{\left(\frac{|x|+|y|}{2}\right)}.$$

Percent error

The **percent error** is a special case of the percentage form of relative change calculated from the absolute change between the experimental (measured) and theoretical (accepted) values, and dividing by the theoretical (accepted) value.

$$\%\ \text{Error} = \frac{|\text{Experimental} - \text{Theoretical}|}{|\text{Theoretical}|} \times 100.$$

The terms "Experimental" and "Theoretical" used in the equation above are commonly replaced with similar terms. Other terms used for *experimental* could be "measured," "calculated," or "actual" and another term used for *theoretical* could be "accepted." Experimental value is what has been derived by use of calculation and/or measurement and is having its accuracy tested against the theoretical value, a value that is accepted by the scientific community or a value that could be seen as a goal for a successful result.

Although it is common practice to use the absolute value version of relative change when discussing percent error, in some situations, it can be beneficial to remove the absolute values to provide more information about the result. Thus, if an experimental value is less than the theoretical value, the percent error will be negative. This negative result provides additional information about the experimental result. For example, experimentally calculating the speed of light and coming up with a negative percent error says that the experimental value is a velocity that is less than the speed of light. This is a big difference from getting a positive percent error, which means the experimental value is a velocity that is greater than the speed of light (violating the theory of relativity) and is a newsworthy result.

The percent error equation, when rewritten by removing the absolute values, becomes:

$$\%\ \text{Error} = \frac{\text{Experimental} - \text{Theoretical}}{|\text{Theoretical}|} \times 100.$$

It is important to note that the two values in the numerator do not commute. Therefore, it is vital to preserve the order as above: subtract the theoretical value from the experimental value and not vice versa.

Percentage change

A **percentage change** is a way to express a change in a variable. It represents the relative change between the old value and the new one.

For example, if a house is worth $100,000 today and the year after its value goes up to $110,000, the percentage change of its value can be expressed as

$$\frac{110000 - 100000}{100000} = 0.1 = 10\%.$$

It can then be said that the worth of the house went up by 10%.

More generally, if V_1 represents the old value and V_2 the new one,

$$\text{Percentage change} = \frac{\Delta V}{V_1} = \frac{V_2 - V_1}{V_1} \times 100.$$

When the variable in question is a percentage itself, it is better to talk about its change by using percentage points, to avoid confusion between relative difference and absolute difference.

Example of percentages of percentages

If a bank were to raise the interest rate on a savings account from 3% to 4%, the statement that "the interest rate was increased by 1%" is ambiguous and should be avoided. The absolute change in this situation is 1 percentage point (4% - 3%), but the relative change in the interest rate is:

$$\frac{4\% - 3\%}{3\%} = 0.333\ldots = 33\frac{1}{3}\%.$$

So, one should say either that the interest rate was increased by 1 percentage point, or that the interest rate was increased by $33\frac{1}{3}\%$.

In general, the term "percentage point(s)" indicates an absolute change or difference of percentages, while the percent sign or the word "percentage" refers to the relative change or difference.

Other change units

Change in a quantity can also be expressed logarithmically using the unit of logarithmic change: the neper (Np). Normalization with a factor of 100, as done for percent, yields the derived unit **centineper** (cNp) which aligns with the definition for percentage change for very small changes:

$$D_{cNp} = 100 \cdot \ln \frac{V_2}{V_1} \approx 100 \cdot \frac{V_2 - V_1}{V_1} = \text{Percentage change when } \left|\frac{V_2 - V_1}{V_1}\right| << 1$$

But using cNp has two additional advantages. First, there is no need to keep track of which of the two quantities, V_1 or V_2, the change is expressed relative to, since, under the conditions of the approximation, the two quantities are nearly the same. Second, an *X* cNp change in a quantity following a *-X* cNp change returns that quantity to its original value. For example, if a quantity doubles, this corresponds to a 69cNp change (an increase). When it halves again, it is a -69cNp change (a decrease.)

Examples

Comparisons

Car M costs $50,000 and car L costs $40,000. We wish to compare these costs. With respect to car L, the absolute difference is $10,000 = $50,000 - $40,000. That is, car M costs $10,000 more than car L. The relative difference is,

$$\frac{\$10,000}{\$40,000} = 0.25 = 25\%,$$

and we say that car M costs 25% *more than* car L. It is also common to express the comparison as a ratio, which in this example is,

$$\frac{\$50,000}{\$40,000} = 1.25 = 125\%,$$

and we say that car M costs 125% *of* the cost of car L.

In this example the cost of car L was considered the reference value, but we could have made the choice the other way and considered the cost of car M as the reference value. The absolute difference is now -$10,000 = $40,000 - $50,000 since car L costs $10,000 less than car M. The relative difference,

$$\frac{-\$10,000}{\$50,000} = -0.20 = -20\%$$

is also negative since car L costs 20% *less than* car M. The ratio form of the comparison,

$$\frac{\$40,000}{\$50,000} = 0.8 = 80\%$$

says that car L costs 80% *of* what car M costs.

It is the use of the words "of" and "less/more than" that distinguish between ratios and relative differences.

Notes

[1] What's a good way to check for *close enough* floating-point equality (http://c-faq.com/fp/fpequal.html)

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- "Percent Difference – Percent Error" (http://www.phy.ilstu.edu/slh/Percent Difference Error.pdf). Illinois State University, Dept of Physics. 2004-07-20. Retrieved 2010-05-05.

External links

- http://www.acponline.org/clinical_information/journals_publications/ecp/janfeb00/primer.htm
- http://books.google.com/books?id=AY7LnYkiLNkC&lpg=PA62&ots=_7s0zd_KtZ&dq=%22relative%20difference%22%20%22absolute%20difference%22&pg=PA61

Sales taxes in the United States

Taxation in the United States

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Sales taxes in the United States are taxes placed on the sale or lease of goods and services in the United States. While no national general sales tax exists, the federal government levies several national selective sales taxes on the sale or lease of particular goods and services. Furthermore, forty-five states, the District of Columbia, and Guam impose general sales taxes that apply to the sale or lease of most goods and some services, and states also may levy selective sales taxes on the sale or lease of particular goods or services. States may also delegate to local governments the authority to impose additional general or selective sales taxes.

Sales tax is calculated by multiplying the purchase price by the applicable tax rate. Tax rates vary widely by jurisdiction and range from less than 1% to over 10%. Sales tax is collected by the seller at the time of sale. Use tax is self assessed by a buyer who has not paid sales tax on a taxable purchase. Unlike the value added tax, a sales tax is imposed only at the retail level. In cases where items are sold at retail more than once, such as used cars, the sales tax can be charged on the same item indefinitely.

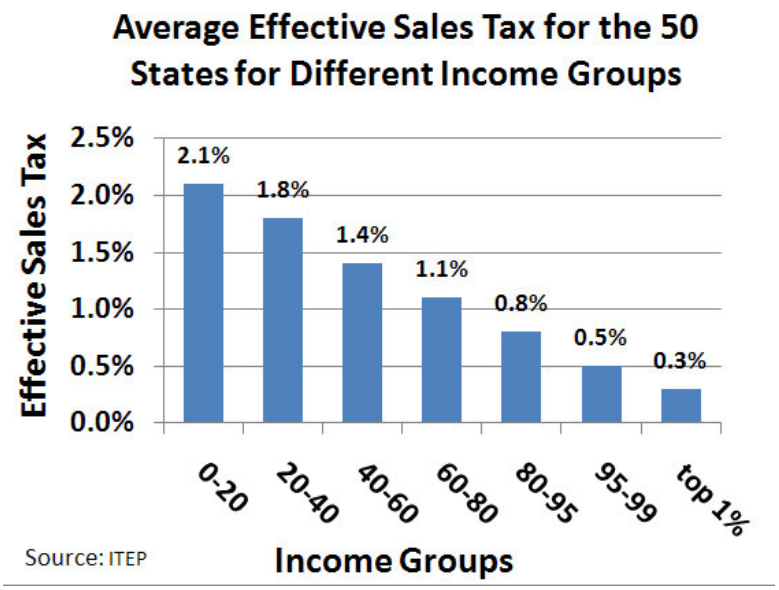
The definition of retail sales and what goods and services are taxable vary among the states. Nearly all jurisdictions provide numerous categories of goods and services that are exempt from sales tax, or taxed at reduced rates. The purchase of goods for further manufacture or for resale is uniformly exempt from sales tax. Most jurisdictions exempt food sold in grocery stores, prescription medications, and many agricultural supplies.

Sales taxes, including those imposed by local governments, are generally administered at the state level. States imposing sales tax either impose the tax on retail sellers or impose it on retail buyers and require sellers to collect it. In either case, the seller files returns and remit the tax to the state. In states where the tax is on the seller, it is customary for the seller to demand reimbursement from the buyer. Procedural rules vary widely. Sellers generally must collect tax from in-state purchasers unless the purchaser provides an exemption certificate. Most states allow or require electronic remittance.

Taxable items

Sales taxes are imposed only on *taxable* transfers of goods or services. The tax is computed as the tax rate times the taxable transaction value. Rates vary by state, and by locality within a state.[3] Not all types of transfers are taxable. The tax may be imposed on sales to consumers and to businesses.[4]

All states exempt certain types of goods from sales and use tax. Those states that tax services tax only certain services. Some states tax certain types of property rentals. Most transfers of intangible property are not subject to sales tax. Definitions become critical.



The average effective sales tax for different income groups of the combined 50 States (2007). The effective tax shown is calculated using a microsimulation model based on the 1990 Public Use Microdata Sample of census records and statistical data from the Internal Revenue Service for undisclosed years.[2]

Taxable sales

Transfers of tangible personal property for cash or the promise to pay cash (sales) are often subject to sales tax, with exceptions.[5] Sales tax does not apply to transfers of real property, though some states impose a real estate transfer or documentary tax on such transfers. All states provide some exemptions from sales tax for wholesale sales, that is, sales for resale.[6] However, some states tax sales for resale through vending machines.[7]

Most states also exempt bulk sales, such as sales of an entire business. Most states exempt from sales tax goods purchased for use as ingredients or parts in further manufacturing. Buyers in exempt sales must follow certain procedures or face tax.

Sales to businesses and to consumers are generally taxed the same, except as noted in the preceding paragraph. Businesses receive no offset to sales tax collection and payment obligations for their own purchases. This differs significantly from value added taxes.

The place and manner of sale may affect whether a sale of particular goods is taxable. Many states tax food for consumption on premises but not food sold for off premises consumption.[8] The use to which goods are put may also affect whether the sale is subject to tax. Goods used as ingredients in manufacturing may avoid tax, where the same goods used as supplies may not.[9]

Rentals

Many states tax rental of tangible personal property. Often the tax is not dependent on the use to which the property will be put.

Florida is the only state to charge sales tax on the rental of real estate, commercial or residential, raising more than a billion dollars a year from the tax. Residential rentals of more than six months are exempted from the tax.

Exempt organizations

Many states exempt charitable, religious, and certain other organizations from sales or use taxes on goods purchased for the organization's use.[10] Generally such exemption does not apply to a trade or business conducted by the organization.[11]

Use tax

The states imposing sales taxes also impose a similar tax on buyers of taxable property or services in those cases where sales tax is not paid. Use taxes are functionally equivalent to sales taxes. The sales and use taxes, taken together, "provide a uniform tax upon either the sale or the use of all tangible personal property irrespective of where it may be purchased."[12] Some states permit offset of sales taxes paid in other states on the purchased goods against use tax in the taxpayer's state.[13]

Taxable value

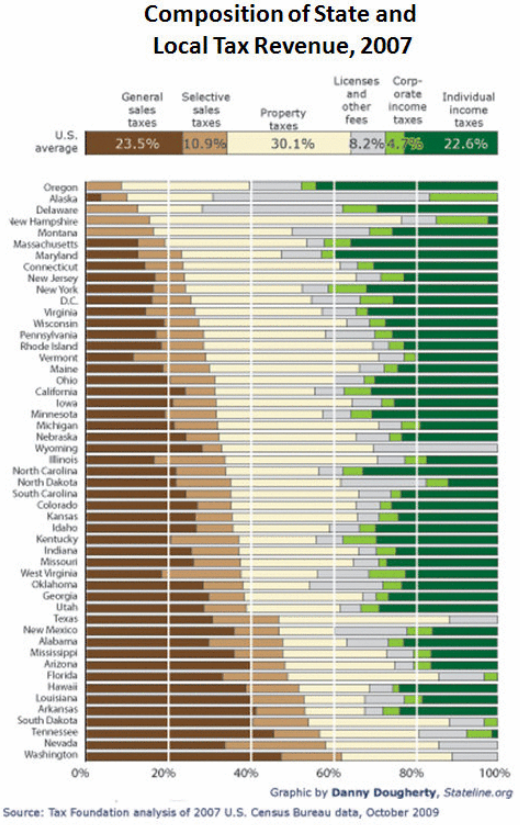
The amount subject to sales tax is generally the net sales price. Such price is generally after any applicable discounts.[14]

Some states exempt a portion of sales or purchase price from tax for some classes of goods.[15]

Taxable goods

No state imposes sales tax on all types of goods. State laws vary widely as to what goods are subject to tax. Food for preparation and consumption in the home is generally not taxable, nor are prescription medications. By contrast, restaurant meals are often taxed.[16]

Many states provide exemptions for some specific types of goods and not for other types. Certain types of foods may be exempt, and certain types taxable, even when sold in a grocery store for home consumption.[17] Lists of what goods are taxable and what are not may be voluminous.[18]



The composition of state and local tax revenues by sales taxes (brown), property taxes (white), licenses and other fees (grey), individual and corporate income taxes (green) in 2007.

Services

Most states tax some services, and some states tax many services. However, taxation of services is the exception rather than the rule.[19] Few states tax the services of a doctor, dentist, or attorney. Services performed in connection with sale of tangible personal property are often taxed. Most states, however, tax services that are an integral part of producing goods, such as printing or cabinet making.[20]

Telecommunications services are subject to a tax similar to a sales tax in most states. Only a few states tax internet access or other information services. Construction services are rarely taxed by states. Materials used in construction of real property may be subject to sales tax to the builder, the subcontractor, or the person engaging the builder, or may be wholly exempt from sales tax.

Intangible property

Most sales tax laws do not apply to most payments for intangible property. Some states tax certain forms of intangible property transfers or licenses. A common transaction subject to sales tax is license of "shrink wrap" software.[21] State courts have often found that numerous transfers of intangible rights are to be considered subject to sales tax where not specifically exempted.[22]

Sales for resale

All states exempt from sales or use tax purchases of goods made for resale in the same form.[23] In many states, resale includes rental of the purchased property. Where the purchased property is not exactly the property resold, the purchase may be taxable. Further, use of the property before sale may defeat the resale exemption.[24] Goods purchased for free distribution may be taxed on purchase in some states, and not in others.[25]

Goods purchased to be used as ingredients in manufacturing tangible personal property are generally not taxable. Purchases of food by a restaurant generally are not taxable in those states that tax sales by restaurants, even though the ingredients are transformed. Steel purchased to be part of machines is generally not taxable. However, supplies consumed by the same businesses may be taxable. Criteria vary widely by state.[26]

Purchase of goods to be provided as part of performance of services may be taxed. Airlines and hotels may be taxed on purchases of food to be provided as part of their services, such as in-flight meals or free breakfast.[27] Where there is a separate charge for such goods, they may be considered purchased for resale.[28]

Distinguishing goods from nontaxable items

Since services and intangibles are typically not taxed, the distinction between a taxable sale of tangible property and a nontaxable service or intangible transfer is a major source of controversy.[29] Many state tax administrators and courts look to the "true object" or "dominant purpose" of the transaction to determine if it is a taxable sale.[30] Some courts have looked at the significance of the property in relation to the services provided.[31] Where property is sold with an agreement to provide service (such as an extended warranty or service contract), the service agreement is generally treated as a separate sale if it can be purchased separately. Michigan and Colorado courts have adopted a more holistic approach, looking at various factors for a particular transaction.

Collection, payment and tax returns

Sales taxes are collected by vendors in most states. Use taxes are self assessed by purchasers. Many states require individuals and businesses who regularly make sales to register with the state. All states imposing sales tax require that taxes collected be paid to the state at least quarterly. Most states have thresholds at which more frequent payment is required. Some states provide a discount to vendors upon payment of collected tax.

Sales taxes collected in some states are considered to be money owned by the state, and consider a vendor failing to remit the tax as in breach of its fiduciary duties. Sellers of taxable property must file tax returns with each jurisdiction in which they are required to collect sales tax. Most jurisdictions require that returns be filed monthly, though sellers with small amounts of tax due may be allowed to file less frequently.

Sales tax returns typically report all sales, taxable sales, sales by category of exemption, and the amount of tax due. Where multiple tax rates are imposed (such as on different classes of property sold), these amounts are typically reported for each rate. Some states combine returns for state and local sales taxes, but many local jurisdictions require separate reporting. Some jurisdictions permit or require electronic filing of returns.

Purchasers of goods who have not paid sales tax in their own jurisdiction must file use tax returns to report taxable purchases. Many states permit such filing for individuals as part of individual income tax returns.

Exemption certificates

Purchasers are required to pay sales tax unless they present the seller with certification that the purchase is exempt from tax (exemption certificate). The certificate must be on a form approved by the state. 38 states have approved use of the Multistate Tax Commission's Uniform Sales and Use Tax Certificate [32].

Exemptions typically fall into two categories: usage based or entity based. Use based exemptions are when an otherwise taxable item or service is used in a manner that has been deemed exempt. The resale exemption is the most common use based exemption. Other use based exemptions could be items or services to be used in manufacturing, research & development, or teleproduction. Entity based exemptions are when the item or service is exempt solely because the purchaser falls into a category the state has granted an exempt status. Exempt entities could be government (federal, state or local), non-profit organizations, religious organizations, tribal governments, or foreign diplomats. Every state decides for themselves which use based and entity based exemption they will grant.

Penalties

Persons required to file sales or use tax returns who do not file are subject to penalties. Persons who fail to properly pay sales and use tax when due are also subject to penalties. The penalties tend to be based on the amount of tax not paid, and vary by jurisdiction.

Tax audits

All states imposing sales taxes examine sales and use tax returns of many taxpayers each year. Upon such audit, the state may propose adjustment of the amount of tax due. Taxpayers have certain rights of appeal, which vary by jurisdiction. Some states require payment of tax prior to judicial appeal, and some states consider payment of tax an admission of the tax liability. Identifying departments that conduct audits in states seems to be near impossible

Constitutional limitations

The United States Constitution generally does not limit the ability of a state to impose a sales or use tax on persons using goods within the state. However, it does impose limitations on each state's ability to tax interstate commerce under the Commerce Clause and the Due Process Clause. States are prohibited from requiring out of state sellers to collect tax unless the seller has established a physical presence in the state sufficient to establish a nexus.[33] Only sellers with at least a minimal connection (nexus) with the state can be required to collect sales tax.

Several state constitutions impose limitations on sales tax. These limitations restrict or prohibit the taxing of certain items, such as food.[34]

By jurisdiction

Sales tax rates and what is taxed vary by jurisdiction. The following table compares taxes on selected classes of goods in the states. Significant other differences apply. Following the table is abbreviated coverage of selected sales tax rates by state.[35]

Summary table

Color	Explanation
	Exempt from general sales tax
	Subject to general sales tax
7%	Taxed at a higher rate than the general rate
3%	Taxed at a lower rate than the general rate
3%+	Some locations tax more
3% (max)	Some locations tax less
> $50	Taxed purchases over $50 (otherwise exempt)
	No state-wide general sales tax

State	General Tax	Total with Max Local Surtax	Groceries	Prepared Food	Prescription Drug	Non-prescription Drug	Clothing
Alabama	4%	11%					
Alaska	0%	7%					
Arizona	5.6%	10.725%					
Arkansas	6.5%	12%	1.5%+				
California	7.5%	10%					
Colorado	2.9%	10%					
Connecticut	6.35%	6.35%					
Delaware	0%	0%					
District of Columbia	6%	6%		10%			
Florida	6%	7.5%		9% (max)			
Georgia	4%	8%	3% (max)				
Guam	4%	4%					
Hawaii	4%	4.5%					
Idaho	6%	8.5%					
Illinois	6.25%	10%	1%+	8.25%+	1%+	1%+	
Indiana	7%	7%		9% (max)			
Iowa[36]	6%	7%					
Kansas	6.15%	9.65%					
Kentucky	6%	6%					
Louisiana	4%	11%					
Maine	5.5%	5.5%		7%			
Maryland	6%	6%					
Massachusetts	6.25%	6.25%		7% (max)			> $175
Michigan	6%	6%					
Minnesota	6.875%	7.875%		10.775% (max)			
Mississippi	7%	7.25%					
Missouri	4.225%	9.35%	1.225%				

Montana	0%	0%					
Nebraska	5.5%	7.5%		9.5% (Omaha)			
Nevada	6.85%	8.1%					
New Hampshire	0%	0%		9%			
New Jersey	7%	7%					
New Mexico	5.125%	8.688%					
New York	4%	8.875%					> $110
North Carolina	4.75%	7.50%	2%	8.50% (max)			
North Dakota [37]	5%	8%					
Ohio	6.5%	8%		Dine-in			
Oklahoma	4.5%	11%					
Oregon	0%	0%					
Pennsylvania	6%	8%					
Puerto Rico	7.0% - 9.0%	7%					
Rhode Island	7%	7%		8%			
South Carolina	6%	9%		10.5%			
South Dakota	4%	6%					
Tennessee	7%	9.75%	5%				
Texas	6.25%	8.25%					
Utah	4.7%	8.35%	4%				
Vermont	6%	7%		9%+			
Virginia	4.3%	6%	2.5%	5.3%+			
Washington	6.5%	9.6%		10% (max)			
West Virginia	6%	7%					
Wisconsin	5%	5.6%					
Wyoming	4%	6%					

Notes:

- These states tax food but give an income tax credit to compensate poor households: HI, ID, KS, OK, SD, and WY.
- Includes statewide local tax of 1.0% in California and 1.0% in Virginia.
- An unspecified state's tax rate may be adjusted annually according to a formula based on balances in the unappropriated general fund and the school foundation fund.[*citation needed*]

Alabama

Alabama has a state general sales tax of 4.00%, plus any additional local taxes which can amount to a combined total sales tax of up to 12 percent in some cities.[38] Alabama is one of several states that do not exempt food from state taxes. The capital of Montgomery has a sales tax of 10%. The state's largest city, Birmingham, has a sales tax of 10 percent. The City of Decatur has a 9% tax in most of its city limits, but has a 10% tax on the small portion of the city that is in Limestone County, due to the higher county tax for Limestone. This only affects about 5 businesses, which are mostly gas stations and sandwich shops.

Alaska

There is no state sales tax in Alaska; however, local governments - which include boroughs, the Alaska equivalent of counties, and municipalities - may levy up to 7.5 percent. As of January 2009, 108 boroughs and municipalities of them do so. Municipal sales taxes are collected in addition to borough sales taxes, if any. Regulations and exemptions vary widely across the state.[39] The two largest cities, Anchorage and Fairbanks, do not charge a local sales tax. The state capital, Juneau, has a 5 percent sales tax rate.

Arizona

Arizona has a transaction privilege tax (TPT) that differs from a "true" sales tax in that it is a gross receipts tax, a tax levied on the gross receipts of the vendor and not a liability of the consumer.[40] (As explained in Arizona Administrative Code rule R15-5-2202,[41] vendors are permitted to pass the amount of the tax on to the consumer, but remain the liable parties for the tax to the state.) TPT is imposed under 16 tax classifications (as of November 1, 2006),[42] with the tax rate most commonly encountered by Arizona consumers (e.g., for retail transactions) set at 6.6 percent.

Cities and counties can add as much as 6 percent to the total rate.[43] Food for home consumption and prescription drugs (including prescription drugs and certain prescribed homeopathic remedies) are two of many items of tangible personal property that are statutorily exempt from the state retail TPT; cities can charge tax on food, and many do. Arizona's TPT is one of the few excise taxes in the country imposed on contracting activities rather than sales of construction materials.[44] Phoenix, the capital and largest city, has a 2% TPT rate.

A law passed in July 2011 created a requirement that Arizona residents declare how much use tax they owe on purchases made from out-of-state online retailers and catalogs.

Arkansas

Arkansas has a state sales tax of 6.00%, plus any additional local taxes; for instance, Little Rock charges a 1.5% city sales tax.

Effective July 1, 2011, Arkansas state sales tax on unprepared food (groceries) was reduced to 1.5% from 2%. Sales taxes on groceries had previously been reduced to 2% from 3% on July 1, 2009 and to 3% from 6% on July 1, 2007. Local sales taxes on groceries remained unchanged.

Here is a quick form to determine how much to charge.

California

California has a base sales tax of 7.50%, and can total up to 9.00% with local sales tax included depending on the city in which the purchase is made. Sales and use taxes in the state of California are collected by the publicly elected Board of Equalization, whereas income and franchise taxes are collected by the Franchise Tax Board.

In general, sales tax is required on all purchases of tangible personal property to its ultimate consumer. Medical devices such as prosthetics and dental implant fixtures are exempt from sales tax with the exception of prosthetic teeth such as dentures, dental orthotics/orthopedic devices, and dental crowns which the state treats as personal property. Unprepared food, bakery items, hot beverages, livestock, crops and seeds, fertilizer used to grow food, certain devices related to alternative energy, and one-time sales are also exempt from sales tax.

Colorado

Colorado's state sales tax is 2.9% with some cities and counties levying additional taxes. Denver's tangibles tax is 3.62%, with food eaten away from the home being taxed at 4%, most unprepared food (groceries) are exempt. There is also a football stadium tax (expired 12/31/11), mass transit tax, and scientific and cultural facilities tax. The total sales tax varies by city and county. Total sales tax on an item purchased in Falcon, Colorado, would be 4.9% (2.9% state, 1% county, and 1% RTA). The sales tax rate in Larimer County is roughly 7.5%. Most transactions in Denver and the surrounding area are taxed at a total of about 8%. The sales tax rate for non food items in Denver is 7.62%. Food & Beverage items total 8.00%, and Rental Cars total 11.25%

Connecticut

Connecticut has a 6.35% sales tax, with no additional local taxes. Most non-prepared food products are exempt, as are most prescription and nonprescription medications, all internet services, all magazine and newspaper subscriptions, and textbooks (for college students only).[45] Also Compact Fluorescent Light Bulbs are tax exempt per Connecticut State Law

Shipping and delivery charges (including charges for U.S. postage) made by a retailer to a customer are subject to sales and use taxes when provided in connection with the sales of taxable tangible personal property or services. The tax applies even if the charges are separately stated and applies regardless of whether the shipping or delivery is provided by the seller or by a third party. No tax is due on shipping and delivery charges in connection with any sale that is not subject to sales or use tax. Shipping or delivery charges related to sales for resale or sales of exempt items are not taxable. Likewise, charges for mailing or delivery services are not subject to tax if they are made in connection with the sale of nontaxable services.[46]

Delaware

Delaware does not assess a sales tax on consumers. The state does, however, impose a tax on the gross receipts of most businesses. Business and occupational license tax rates range from 0.096 percent to 1.92 percent, depending upon the category of business activity. However, it does charge a 3.75% 'document' fee on vehicle registrations.

District of Columbia

Washington, D.C., has a sales tax rate of 5.75% as of October 1, 2013. The tax is imposed on sale of tangible personal property and selected services. A 10% tax is imposed on liquor sold for off premises consumption, 10% on restaurant meals (including carry-out) and rental cars, 18% on parking, and 14.5% on hotel accommodations. Groceries, prescription and non-prescription drugs, and residential utilities services are exempt from the District's sales tax.

The District once had two sales tax holidays each year, one during "back-to-school" and one immediately preceding Christmas. The "back to school"" tax holiday was repealed on May 12, 2009.[47]

On January 1, 2010, the District began levying a 5 cent per bag tax on each bag provided by a retailer at the point of sale, if that retailer sells food or alcohol. The retailer retains one cent of the tax, or two cents if they offer a refund to customers for bringing their own bags. The remaining three or four cents goes to the District's Anacostia River cleanup fund.

Florida

Florida has a general sales tax rate of 6%. Miami-Dade County, like most Florida counties, has an additional county sales surtax. Miami-Dade's surtax is 1%.[48] The tax is imposed on the sale or rental of goods, the sale of admissions, the lease, license, or rental of real property, the lease or rental of transient living accommodations, and the sale of a limited number of services such as commercial pest control, commercial cleaning, and certain protection services. There are a variety of exemptions from the tax, including groceries and prescriptions.[49]

Sales tax and discretionary sales surtax are calculated on each taxable transaction. Florida uses a “bracket system” for calculating sales tax when the transaction falls between two whole dollar amounts. Multiply the whole dollar amount by the tax rate (6 percent plus the county surtax rate) and use the bracket system to figure the tax on the amount less than a dollar. The Department of Revenue has rate tables (Form DR-2X) to assist residents.

A "discretionary sales surtax" may be imposed by the counties of up to 1.5%, charged at the rate of the destination county (if shipped). This is 1% in most counties, 0.5% in many, 1.5% in a few such as Leon, and 0.25% in one county (Alachua). A few counties have no additional surtax. Most have an expiration date, but a few do not. Only the first $5,000 of a large purchase is subject to the surtax rate.[50] Most counties levy the surtax for education or transportation improvements.

There are annual sales tax holidays, such as a back-to-school holiday on clothing, books, and school supplies under a certain price, as well as one in June 2007 to promote hurricane preparedness. The 2008 Legislators did not enact any sales tax holidays.

Florida also permits counties to raise a "tourist development tax" of up to an additional 13% for stays of 6 months or less on any hotel, apartment hotel, motel, resort motel, apartment, apartment motel, rooming house, mobile home park, recreational vehicle park, condominium, or timeshare resort.[51]

In May 2010 Florida passed a law that capped sales tax on boats to a maximum of $18,000, regardless of the purchase price. This was to encourage owners not to leave the State after purchase or to flag "offshore" which most owners were doing prior to the passing of this law. As a result the Florida Dept. of Revenue has seen a dramatic increase in sales tax revenues from the sale of boats and the State ranks #1 in boat registrations.

Georgia

Georgia has had a 4% state sales tax rate since April 1, 1989, when it was raised from 3%.[52] Groceries are exempt from the state sales tax, but still subject to tax by the local sales tax rate. Counties in Georgia may impose local sales tax of 1%, 2%, or 3%, consisting of up to three 1% local-option sales taxes (out of a set of five) as permitted by Georgia law. These include a special-purpose tax (SPLOST) for specific projects, a general-purpose LOST, a homestead exemption (HOST), and one for public schools (ELOST) which can be put forth for a referendum by the school board instead of the county commission (in cooperation with its city councils) as the other taxes are. Also, the city of Atlanta imposes an additional 1% municipal-option sales tax (MOST), as allowed by special legislation of the Georgia General Assembly. This is[53] for fixing its water and sewerage systems.[54]

As of July 2012,[55] total sales tax rates in Georgia are 3% for groceries and 7% for other items in 149 of its 159 counties. A few counties charge only 2% local tax (6% total on non-grocery items), and four partially exempt groceries from the local tax by charging 2% on food, and 3% (7% total) on other items. Just one county (Whitfield) charged only 1% local tax. Fulton and DeKalb counties charge 1% for MARTA, and adjacent metro Atlanta counties may do so by referendum if they so choose. For the portions of Fulton and DeKalb within the city of Atlanta, the total is at 4% and 3% respectively on groceries and 8% on other items due to the MOST.[56]

Similar to Florida and certain other states, Georgia used to have two sales-tax holidays per year, starting in 2002. One was for back-to-school sales the first weekend in August, but sometimes starting at the end of July. A second usually occurred in October, for energy-efficient home appliances with the Energy Star certification. There were no sales-tax holidays in 2010 and 2011, but they were reinstated in 2012.[57]

Starting in 2012, Georgia now imposes a title-transfer tax on all automobiles sold in the state, even for sales from one private owner directly to another. This is based on the price of the vehicle, effectively making it a sales tax. It still continues to collect the annual *ad valorem* taxes on existing owners, resulting in double taxation, which the Georgia Department of Revenue reports has significantly increased vehicle tax collections as of August 2013.

Georgia has many exemptions available to specific businesses and industries. To identify potential exemptions, businesses and consumers must research the laws and rules for sales and use tax and review current exemption forms.[58]

Guam

Guam has no general sales tax imposed on the consumer with the exception of admissions, use, and hotel occupancy taxes; however, businesses must pay 4% tax on their monthly gross income. There are no separate municipal, county, school district or improvement district taxes.

Use tax is 4% on non-exempt personal property imported to Guam. Hotel tax is 11% of daily room rate. Alcoholic beverage tax varies depending on the beverage. Additionally there are tobacco taxes, real property taxes, amusement taxes, recreational facility taxes, and liquid fuel taxes.

Hawaii

Hawaii does not have a sales tax *per se*, but it does have a gross receipts tax (called the General Excise Tax) and a Use Tax which apply to nearly every conceivable type of transaction (including services), and is technically charged to the business rather than the consumer. Hawaii law allows businesses to pass on the tax to the consumer in similar fashion to a sales tax.

Unlike other states, rent, medical services and perishable foods are subject to the excise tax. Also, unlike other states, businesses are not required to show the tax separately on the receipt, as it is technically part of the selling price. Most retail businesses in Hawaii, however, do list the tax as a separate line item. 4.0% is charged at retail with an additional 0.5% surcharge in the City and County of Honolulu (for a total of 4.5% on Oahu sales), and 0.5% is charged on wholesale. However, the state also allows "tax on tax" to be charged, which effectively means a customer can be billed as much as 4.166% (4.712% on Oahu). The exact dollar or percentage amount to be added must be quoted to customers within or along with the price. The 0.5% surcharge on Oahu was implemented to fund the new rail transport system. As with sales tax in other states, nonprofit organizations may apply for an exemption from the tax.

Hawaii also imposes a “Use tax” on businesses that provide services that are “LANDED” In Hawaii. One example is: A property owner in Hawaii contracts with a mainland architect to design their Hawaii home. Even though the architect perhaps does all of their work in a mainland location the architect needs to pay the State of Hawaii a 4% Use tax on the architect’s fee because the designed house is located in Hawaii. That holds true even if the house is never built. The tax is on the produced product which is the design and provided building plans.

Taking that example a step further. A major retail chain on the US mainland contracts with a mainland architect to design a store in Hawaii. Because the store is located in Hawaii the architect is required to pay the State of Hawaii 4% on the entire fee they charge the retail chain to design the Hawaii store.

Idaho

Idaho has a 6.0% state sales tax. Some localities levy an additional local sales tax.[59]

Illinois

Illinois' sales and use tax scheme includes four major divisions: Retailers' Occupation Tax, Use Tax, Service Occupation Tax and the Service Use Tax.[60] Each of these taxes is administered by the Illinois Department of Revenue. The Retailers' Occupation Tax is imposed upon persons engaged in the business of selling tangible personal property to purchasers for use or consumption. It is measured by the gross receipts of the retailer. The base rate of 6.25% is broken down as follows: 5% State, 1% City, 0.25% County. Local governments may impose additional tax resulting in a combined rate that ranges from the State minimum of 6.25% to 9.00% as of May 2013.[61][62]

Springfield charges 8.00% total (including state tax). A complementary Use Tax is imposed upon the privilege of using or consuming property purchased anywhere at retail from a retailer. Illinois registered retailers are authorized to collect the Use Tax from their customers and use it to offset their obligations under the Retailers' Occupation Tax Act. Since the Use Tax rate is equivalent to the corresponding Retailers' Occupation Tax rate, the amount collected by the retailer matches the amount the retailer must submit to the Illinois Department of Revenue. The combination of these two taxes is what is commonly referred to as "sales tax." If the purchaser does not pay the Use Tax directly to a retailer (for instance, on an item purchased from an Internet seller), they must remit it directly to the Illinois Department of Revenue.[63]

The Service Occupation Tax is imposed upon the privilege of engaging in service businesses and is measured by the selling price of tangible personal property transferred as an incident to providing a service. The Service Use Tax is imposed upon the privilege of using or consuming tangible personal property transferred as an incident to the provision of a service. An example would be a printer of business cards. The printer owes Service Occupation Tax on the value of the paper and ink transferred to the customer in the form of printed business cards. The serviceperson may satisfy this tax by paying Use Tax to his supplier of paper and ink or, alternatively, may charge Service Use Tax to the purchaser of the business cards and remit the amount collected as Service Occupation Tax on the serviceperson's tax return. The service itself, however, is not subject to tax.

Qualifying food, drugs, medicines and medical appliances[64] have sales tax of 1% plus local home rule tax depending on the location where purchased. Newspapers and magazines are exempt from sales tax as are legal tender, currency, medallions, bullion or gold or silver coinage issued by the State of Illinois, the government of the United States of America, or the government of any foreign country.

The city of Chicago has one of the highest total sales tax of all major U.S. cities (9.25%).[65] It was previously higher (10.25%), however, it was reduced when Cook County lowered its sales tax by 0.5% in July 2010, another 0.25% in January 2012, and another 0.25% in January 2013.[66] Chicago charges a 2.25% food tax on regular groceries and drug purchases, and has an additional 3% soft drink tax (totaling 12.5%). An additional 1% is charged for prepared food and beverage purchases in the Loop and nearby neighborhoods (the area roughly bordered by Diversey Parkway, Ashland Avenue, the Stevenson Expressway, and Lake Michigan).

Illinois requires residents who make purchases online or when traveling out-of-state to report those purchases on their state income tax form and pay use tax.

Indiana

Indiana has a 7% state sales tax. The tax rate was raised from 6% on April 1, 2008, to offset the loss of revenue from the statewide property tax reform, which is expected to significantly lower property taxes. Untaxed retail items include medications, water, ice and unprepared, raw staple foods or fruit juices. Many localities, inclusive of either counties or cities, in the state of Indiana also have a sales tax on restaurant food and beverages consumed in the restaurant or purchased to go.

Revenues are usually used for economic development and tourism projects. This additional tax rate may be 1% or 2% or other amounts depending on the county in which the business is located. For example, in Marion County, the sales tax for restaurants is 9%. There is an additional 2% tax on restaurant sales in Marion County to pay for Lucas Oil Stadium and expansion of the Indiana Convention Center.

Iowa

Iowa has a 6% state sales tax and an optional local sales tax of 1% imposed by most cities and unincorporated portions of most counties, bringing the total up to a maximum of 7%. There is no tax on most unprepared food. The Iowa Department of Revenue provides information about local option sales taxes,[67] including sales tax rate lookup. Iowa also has sales tax on services when rendered, furnished, or performed.[68]

Kansas

Kansas has a 6.15% state sales tax as of July 1, 2013. More than 700 jurisdictions within the state (cities, counties, and special districts) may impose additional taxes. For example, in the capital city of Topeka, retailers must collect 6.15% for the state, 1.15% for Shawnee County, and 1% for the city, for a total rate of 8.45%. As of July 2010, the highest rate was 8.85%, in Douglas County. Actually right now in Junction City, Geary County, the total sales tax is 9.9% October 2013.

Kentucky

Kentucky has a 6% state sales tax. Most staple grocery foods are exempt. Alcohol sales were previously exempt until April 1, 2009, when a 6% rate was applied to this category as well.

Louisiana

Louisiana has a 4% state sales tax: 3.97% to sales tax and .03% to Louisiana tourism district.[69] The state sales tax is not charged on unprepared food. There are also taxes on the parish (county) level and some on the city levels, Baton Rouge has a 5% sales tax. Parishes may add local taxes up to 5%, while local jurisdictions within parishes may add more. In Allen Parish, the combined sales tax is up to 10% (0.7% for Parish Council, 3% for School Board, 1% to 1.3% for City/Town). Louisiana also bids out sales tax audits to private companies, with many being paid on a percentage collected basis.

New Orleans (coterminous with Orleans Parish) collects the maximum 5% tax rate for a total of 9% on general purpose items. This 5% is also charged on groceries.

Maine

Maine has a 5.5% general, service provider and use tax (raised, temporarily, from 5% on October 1, 2013).[70] The tax on lodging and prepared food is 8% and short term auto rental is 10%. These are all generally known as "sales tax".

Maryland

Maryland has a 6% state sales and use tax (raised from 5% in 2007) as of January 3, 2008, with exceptions for medicine,[71] residential energy, and most non-prepared foods.[72][*citation needed*] While most goods are taxed, many services (e.g., repair,[73] haircuts, accounting) are not. Maryland's sales tax includes Internet purchases and other mail items such as magazine subscriptions.[*citation needed*] Maryland has a "back-to-school" tax holiday on a limited number of consumer items.[*citation needed*] On July 1, 2011 the selective sales tax on alcohol was raised from 6% to 9%.[*citation needed*]

On January 1, 2012, Montgomery County began levying a 5 cent per bag tax on plastic bags provided by retailers at the point of sale, pickup, or delivery. Four cents of this tax goes to the county's water quality fund, and one cent is returned to the retailer.

Massachusetts

Massachusetts has a 6.25% state sales tax (raised from 5% in 2009). Meal tax on prepared food is 6.25% statewide, however in some towns, voters chose to add 0.75% (raising the tax to 7%), with that incremental revenue coming back to the town. Sales tax on liquor was repealed in a 2010 referendum vote. Sales of individual items of clothing costing $175 or less are generally exempt; on individual items costing more than $175, sales tax is due only on the amount over $175.[74] On January 16, 2013, governor Deval Patrick proposed to cut the sales tax to 4.5% in his State of the Commonwealth address, although this simply proposes the idea and nothing has been made official yet.

Michigan

Michigan has a 6% sales tax (raised from 4% in 1994). Michigan has a use tax of 6%, which is a tax that is applied to items that were brought into Michigan but not bought there, and of which taxes were not paid to the state in which the item was bought in. The tax is supposed to be paid when filing a resident's annual income tax.[75]

A service tax was approved in September 2007, effective December 1, 2007, allowing certain services to be taxed. The services tax was repealed the same day it went into effect. There is no local sales tax in Michigan. There is now a Michigan sales tax on the total amount of online orders, meaning that shipping charges are now taxed. Food, periodicals, and prescription drugs are not taxed. Food served in restaurants and other 'prepared food' is taxed at the general sales tax rate.

Minnesota

Minnesota currently has a 6.875% statewide sales tax. The statewide portion consists of two parts: a 6.5% sales tax with receipts going to the state General Fund, and a 3/8 of 1 percent tax going to arts and environmental projects. The 3/8 of 1 percent tax was passed by a statewide referendum on Nov. 4, 2008, and went into effect on July 1, 2009. Generally, food (not including prepared food, some beverages such as pop, and other items such as candy) and clothing are exempt from the sales tax. Prescription drugs are also exempt.[76]

Local units of government may, with legislative approval, impose additional general sales taxes. As of July 1, 2008, an additional 0.25% Transit Improvement tax [77] was phased in across five counties in the Minneapolis-St. Paul metropolitan area for transit development. These counties are Hennepin, Ramsey, Anoka, Dakota, and Washington. A 0.15% sales tax is imposed in Hennepin County to finance the Minnesota Twins' new Target Field. Several cities impose their own citywide sales tax: Saint Paul (0.5%), Minneapolis (0.5%), Rochester (0.5%), and Duluth (1%).

These additional taxes increase the total general sales tax rates to 7.875% in Duluth, 7.775% in Minneapolis, 7.625% in Saint Paul, and 7.375% in Rochester.

In addition to general sales taxes, local units of government can, again with legislative approval, impose sales taxes on certain items. Current local option taxes include a "lodging" tax in Duluth (3%), Minneapolis (3%), and Rochester (4%), as well as served "food and beverage" tax in Duluth (2.25%).

Alcohol is taxed at an additional 2.5% gross receipts tax rate above the statewide 6.875% sales tax rate, for a total rate of 9.375%, not including any applicable local taxes. This totals 10.375% in Duluth, 10.275% in Minneapolis, 10.125% in Saint Paul, and 9.875% in Rochester.

Mississippi

Mississippi has a 7% state sales tax. Cities and towns may implement an additional tourism tax on restaurant and hotel sales. The city of Tupelo has a 0.25% tax in addition to other taxes. Restaurant and fast food tax is 9%. The city of Hattiesburg also has a 9% sales tax on restaurant and fast food.

Missouri

Missouri imposes a sales tax upon all sales of tangible personal property, as well as some "taxable services";[78] it also charges a use tax for the "privilege of storing, using or consuming within this state any article of tangible personal property."[79] The state rate, including conservation and other taxes, is 4.225%, and counties, municipalities, and other political subdivisions charge their own taxes.[80] Those additional local taxes combined with "community improvement district," "transportation development district," and "museum district" taxes can result in merchandise sales taxes in excess of 10%.[81] The state sales tax rate on certain foods is 1.225%.[82]

Missouri provides several exemptions from sales tax, such as purchases by charitable organizations or some common carriers (as opposed to "contract carriers").[83] Missouri also excludes some purchases from taxation on the grounds that such sales are not sales at retail; these include sales to political subdivisions.[84] The Supreme Court of Missouri in August, 2009, ruled that when a sale is excluded from taxation - as opposed to exempt from taxation - the seller must self-accrue sales tax on its purchase of the goods and remit the tax on such purchases it made.[85] This decision was reversed by two similar - but not identical - statutes added during the 2010 general assembly's regular session.

Although the purchaser is obligated to pay the tax, the seller is obligated to remit the tax, and when the seller fails to remit, the obligation to pay falls on him. As compensation for collecting and remitting taxes, and as an incentive to timely remit taxes, sellers may keep two percent of all taxes collected each period.[86] There are two exceptions to the general rule that the seller must pay the sales tax when he or she fails to collect it.

First, no sales tax is due upon the purchase of a motor vehicle that must be titled. Instead, the purchaser pays the tax directly to the Department of Revenue within one month of purchase. As long as the vehicle is taken out of state within that first month of purchased and titled elsewhere, no tax is due in Missouri. Second, if the purchaser presents an exemption certificate to the buyer at the time of sale, then the purchaser may be assessed taxes on the purchases if the certificate was issued in bad faith.

Montana

Montana does not have a state sales tax but some municipalities which are big tourist destinations, such as Whitefish, Red Lodge, Big Sky, and West Yellowstone, have a small sales tax (up to 3%). Hotels, campgrounds and similar lodging charge a "lodging and usage tax", usually at the rate of 7%. Rental car companies charge a 4% tax on the base rental rate.

Nebraska

Nebraska has a 5.5% state sales tax from which groceries are exempt. Municipalities have the option of imposing an additional sales tax of up to 1.5%, resulting in a maximum rate of 7.0%. Specific tax rates per counties are available on the web.[87] Omaha also has a 2.5% tax on prepared food and drink.[88]

Nevada

Nevada's state sales tax rate is 6.85%. Counties may impose additional rates via voter approval or through approval of the Legislature; therefore, the applicable sales tax will vary by county from 6.85% to 8.1% in Clark County. Clark County, which includes Las Vegas, imposes four separate county option taxes in addition to the statewide rate - 0.25% for flood control, 0.50% for mass transit, 0.25% to fund the Southern Nevada Water Authority, and 0.25% for the addition of police officers in that county. In Washoe County (which includes Reno), the sales tax rate is 7.725%, due to county option rates for flood control, the ReTRAC train trench project, mass transit, and an additional county rate approved under the Local Government Tax Act of 1991.

For travelers to Las Vegas, note that the lodging tax rate in unincorporated Clark County, which includes the Las Vegas Strip, is 12%. Within the boundaries of the cities of Las Vegas and Henderson, the lodging tax rate is 13%.

New Hampshire

Sales tax in New Hampshire is limited to a 9% tax on prepared meals, 9% on hotel and similar room rental for less than 185 days, 9% on motor vehicle rentals, and 7% on telecommunications services. A 1.5% transfer tax is levied on real estate sales. Taxable meals exclude food and beverages for consumption off premises, but catered and restaurant meals are taxable.

New Hampshire also imposes excise taxes on gasoline tax at $0.196 per gallon, cigarettes at $1.78 per pack, beer at $0.30 per gallon, and electricity at $0.00055 per kilowatt-hour.[89]

New Jersey

The state of New Jersey's sales and use tax rate is 7%. However, there are exceptions to this statewide rate. In Urban Enterprise Zones, UEZ-impacted business districts, and in both Salem County and Cumberland County, sales tax may be charged at 3.5% (50% of the regular rate) on certain items. In addition, local sales taxes are imposed on sales of certain items sold in Atlantic City and Cape May County.[90]

A full list of Urban Enterprise Zones is available on the State of New Jersey web site.[91]

New Jersey does not charge sales tax on unprepared food (except certain sweets and pet food), household paper products, medicine, and clothing. New Jersey does not charge sales tax on goods purchased for resale or on capital improvements but does charge sales tax on certain services. See the NJ Division of Taxation website at: http://www.state.nj.us/treasury/taxation/su.shtml

New Jersey does not charge sales tax on gasoline, but gasoline is subject to a $0.145/gallon excise tax.

Sales of clothing and accessories that are made of fur from the hide or pelt of an animal that is valued at $500 or more are subject to a 6% Fur Clothing Gross Receipts Tax.

New Mexico

New Mexico imposes a gross receipts tax of 5% on most retail sales or leasing of property or performance of services in New Mexico. The tax is imposed on the seller but it is common for the seller to pass the tax on to the purchaser. The state rate is 5.125%. Municipalities may assess an additional gross receipts tax, resulting in rates between 5.375% and 8.8625%.[92] Numerous specific exemptions and deductions apply.[93]

New York

The default sales tax rate in New York is 7%, of which 4% is levied by the state. All counties, by default, are authorized to collect a 3% sales tax on top of the state levy; under the state's home rule laws, counties and other local municipalities may only levy a higher sales tax if it is approved by the New York State Legislature, and this approval must be repeated every two years. As of 2012, all but five counties in New York charge a higher sales tax rate than the default.[94] The combined sales tax in Utica, for example, is 8.75%. In New York City, total sales tax is 8.875%, which includes 0.375% charged in the Metropolitan Commuter Transportation District (MCTD).

On September 1, 2007, New York State eliminated the 4% state sales tax on all clothing and shoes if the single item is priced under $110. Most counties and cities have not eliminated their local sales taxes on clothing and shoes. There are however, 5 cities (most notably New York City) and 11 counties (not counting the five counties which make up New York City: New York, Queens, Kings, Richmond, and Bronx counties) that have done so.

The counties where the year-round exemption applies are Broome, Chautauqua, Chenango, Columbia, Delaware, Dutchess, Greene, Hamilton, Madison (outside the city of Oneida), Rensselaer, Tioga, and Wayne. The cities where the year-round exemption will apply include Binghamton, Gloversville, New York City, Norwich, Olean, and Sherrill. New York also exempts college textbooks from sales tax.

Since June 1, 2008, when products are purchased online and shipped into New York State, some retailers must charge the tax amount appropriate to the locality where the goods are shipped, and in addition, must also charge the appropriate tax on the cost of shipping and handling. The measure states that any online retailer that generates more than $10,000 in sales via in-state sales affiliates must collect New York sales tax. The cumulative gross receipts from sales to New York customers as a result of referrals by all of the seller’s resident representatives total more than $10,000 during the preceding four quarterly sales tax periods.

From October 1, 2010 to March 31, 2011, statewide sales and use tax exemption for clothing and footwear sold for less than $110 was eliminated. For New York City, this meant articles of clothing costing less than $110 were charged 4.375% tax.[95] A state sales tax exemption for clothing and footwear under $55 was reinstated from April 1, 2011 through March 31, 2012. The original ($110) exemption was reinstated after March 31, 2012.[96]

North Carolina

North Carolina has a state-levied sales tax of 4.75%, effective July 1, 2011, with most counties adding an additional 2% tax, for a total tax of 6.75% in 76 of the 100 counties. Mecklenburg County levies an additional 0.5% tax, which is directed towards funding the light rail system, for a total of 7.25% and the sales tax in a few other counties is 7%. Durham County imposes an additional 0.5% tax onto the 7% rate for funding public transportation, making the total rate 7.5%.[97]

There is a 37.5¢ tax per gallon on gas, a 45¢ tax per pack of cigarettes, a 79¢ tax per gallon on wine, and a 53¢ tax per gallon on beer. Most non-prepared food purchases are taxed at a uniform county tax rate of 2%. Alcohol and certain other goods are taxed at a "combined rate" of 7%, which includes both state tax and a 2.25% county tax. Candy, soft drinks, and prepared foods are taxed at the full combined 6.75%-7.5% rate, with some counties levying an additional 1% tax on prepared foods. For the benefit of back-to-school shoppers, there is a sales tax holiday on the first Friday in August through the following Sunday which includes school supplies, school instructional materials, clothing, footwear, sports and recreation equipment, and computers and computer accessories.

North Dakota

North Dakota has a 5% state sales tax for general sales, but varies depending on the category (5%, 7%, 3% and 2%).[98] These additional taxes increase the total general sales tax rates to 7.5% in Fargo and Valley City and Pembina; 7.25% in Grafton; 7.0% in Minot; 6.75% in Grand Forks; 6.5% in West Fargo; Dickinson and Williston; and 6.0% in Bismarck and Mandan.

Ohio

Ohio has a 5.75% state sales tax.[] Counties may levy a permissive sales tax of from 0.25% up to 1.5% and transit authorities, mass transit districts usually centered on one primary county, may levy a sales tax of from 0.25% up to 1.5%. Cuyahoga County has the highest statewide sales tax rate (8%). Tax increments may not be less than 0.25%, and the total tax rate, including the state rate, may not exceed 8.75%. County permissive taxes may be levied by emergency resolution of the county boards of commissioners. Transit authority taxes must and county permissive taxes may be levied by a vote of the electors of the district or county.

Shipping and handling charges are also taxable. Ohio law requires virtually every type of business to obtain an Ohio Sales Tax Certificate Number. If someone sells goods on eBay or the internet and ships them to someone in the state they reside, then they must collect sales tax from the buyer and pay the collected tax to the state on a monthly or quarterly basis. If someone sells less than $4 million in annual sales, they do not have to collect or pay sales tax on out-of-state sales.

Ohio Sales Tax Resale Certificate Example: If living in Ohio and selling or shipping something to someone else in Ohio, then one must collect and pay sales tax to the State of Ohio. But if selling the same item to someone outside the State of Ohio, one need not charge sales tax, but must report the exempt tax sale to the State of Ohio. Ohio also has a gross receipts tax called the Commercial Activity Tax (CAT) [99] that is applicable only to businesses but shares some similarities to a sales tax. "Food for human consumption off the premises where sold" is exempt from sales tax, with the exception of sodas and alcoholic beverages which are taxed at the full rate.[100]

Oklahoma

Oklahoma has a 4.5% sales tax rate. Cities have an additional sales tax which varies, but is generally 3-4% resulting in a total sales tax rate of 7.5% to 8.5%.[101]

Oregon

Oregon has no statewide sales tax, although local municipalities may impose sales taxes if they choose to do so, such as Ashland, which imposes a 5% prepared food tax.[102]

Pennsylvania

Pennsylvania has a 6% sales tax rate. Allegheny County has local sales tax of 1% on top of the PA sales tax rate that totals 7%. Philadelphia County has a local sales tax of 2% on top of the PA sales tax rate that totals 8%, which became effective October 8, 2009.

Food, most clothing, and footwear are among the items most frequently exempted.[103] However, taxed food items include soft drinks and powdered mixes, sports drinks, hot beverages, hot prepared foods, sandwiches, and salad bar meals, unless these items are purchased with food stamps. Additionally, catering and delivery fees are taxed if the food itself is taxed.

Additional exemptions include internet service,[104] newspapers, textbooks, disposable diapers, feminine hygiene products, toilet paper, wet wipes, prescription drugs, many over-the-counter drugs and supplies, oral hygiene items (including toothbrushes and toothpaste), contact lenses and eyeglasses, health club and tanning booth fees, burial items (like coffins, urns, and headstones), personal protective equipment for production personnel, work uniforms,

veterinary services, pet medications, fuel for residential use (including coal, firewood, fuel oil, natural gas, wood pellets, steam, and electricity), many farming supplies and equipment, ice,[105] and tea[106] (including powdered, hot, cold, and flavored).

The taxability of alcoholic beverages is slightly complicated. In Pennsylvania, alcohol is sold to businesses and consumers through the Pennsylvania Liquor Control Board (PLCB). The PLCB always charges sales tax directly to the purchasing entity. Therefore, if a consumer purchases alcohol in PLCB stores, the sales tax is assessed at the point of purchase, but if a consumer purchases alcohol at a licensed business (such as a bar or restaurant), the sales tax is not applied because it had already been paid when the business purchased the alcohol from the PLCB. The PLCB charges an additional 18% levy on liquor and wine, but this tax is always included in the price regardless of the purchasing location. Beer is subject to an excise of $0.08 per gallon.

Puerto Rico

Puerto Rico has a 5.5% commonwealth sales tax that applies to both products and services with few exemptions (including items such as unprocessed foods, prescription medicines and business-to-business services). Additionally, most municipalities have a city sales tax of 1.5% for a total of 7%. Some items that are exempt from commonwealth sales tax, specifically unprocessed foods, may still be subject to the city sales tax in the municipalities.

Rhode Island

Rhode Island has a state sales tax of 7%. The rate was raised from 5% to 6% as a temporary measure in the 1970s, but has not since been lowered. Rhode Island raised its sales tax from 6% to 7% in the early 1990s to pay for the bailout of the state's failed credit unions. The change was initially proposed as a temporary measure, but was later made permanent. Other taxes may also apply, such as the state's 1% restaurant tax. Many items are exempt from the state sales tax, e.g., food (excluding single serve items), prescription drugs, clothing and footwear, newspapers, coffins, and original artwork.

South Carolina

South Carolina has a 6% state sales tax but when combined with local, county and hospitality taxes South Carolina has a maximum sales tax of 10.5%.

In Charleston, the tax rate equals 10.5% with state tax, county tax, local option tax, and the hospitality tax. The City of Myrtle Beach states that mixed liquor drinks can have taxes added as high as 16.5%.

As of June 1, 2007 counties and some cities may impose an additional 1% to 3% sales tax. As of mid-2005, 35 of 46 counties do so. Restaurants may also charge an extra 1-2% tax on prepared food (fast food or take-out) in some places. The state's sales tax on unprepared food disappeared completely November 1, 2007. There is a cap of $300 on sales tax for most vehicles.

Additionally, signs posted in many places of business inform that South Carolina residents over the age of 85 are entitled to a 1% reduction in sales tax.

For the benefit of back-to-school shoppers, there is a sales tax holiday on the first Friday in August through the following Sunday which includes school supplies, school instructional materials, clothing, footwear, sports and recreation equipment, and computers and computer accessories.

South Dakota

South Dakota has a 4% state sales tax, plus any additional local taxes. An additional 1.5% sales tax is added during the summer season on sales occurring in tourism-related businesses and dedicated to the state's office of tourism.

City governments are allowed a maximum of 2% sales tax for use by the local government. Tribal governments are allowed to charge a higher local government tax rate, as they have a special agreement with the State of South Dakota.

While city governments are not restricted to a 2% tax, they can impose a gross receipts tax on things like lodging, alcohol, restaurants, and admissions. These gross receipts are passed on by the business as a tax and could be considered a sales tax.

Tennessee

Tennessee charges 7% on most items. Groceries were taxed at 5.5% as of January 1, 2008, through June 30, 2012. As of July 1, 2013, grocery tax was reduced to 5%. Counties also tax up to 2.75% in increments of 0.25%. Most do so around 2.25%; thus, the majority of TN taxpayers pay 9.25%. If a county does not charge the maximum, its cities can charge and keep all or part of the remainder. Several cities are in more than one county, but none charge a city tax, thus paying only the county taxes.[107]

Texas

The Texas state sales and use tax rate is 6.25%, but local taxing jurisdictions (cities, counties, special purpose districts, and transit authorities, but specifically not including school districts) may also impose sales and use taxes up to 2% for a total of 8.25%.[108] The main items exempt from sales tax include medicines (prescription and over-the-counter), food and food seeds (but prepared food, such as from a restaurant, is subject to sales tax).[109]

Motor vehicle and boat sales are taxed at only the 6.25% state rate; there is no local sales and use tax on these items. In addition, a motor vehicle or boat purchased outside the state is assessed a use tax at the same rate as one purchased inside the state. The sales tax is calculated on the greater of either the actual purchase price or the "standard presumptive value" of the vehicle, as determined by the state, except for certain purchases (mainly purchases from licensed dealers or from auctions).

Lodging rates are subject to a 6% rate at the state level, with local entities being allowed to charge additional amounts. For example, the city of Austin levies a 9% hotel/motel tax, bringing the total to 15%, trailing only Houston for the highest total lodging tax statewide, at 17%. Lodging for travelers on official government business is specifically exempt from tax but the traveler must submit an exemption form to the hotel/motel and provide proof of official status.

If merchants file and pay their sales and use tax on time, they may subtract 1/2 percent of the tax collected as a discount, to encourage prompt payment and to compensate the merchant for collecting the tax from consumers for the state.

Texas provides one sales tax holiday per year (generally in August prior to the start of the school year, running from Friday to Sunday of the designated weekend). Clothing less than $100 (except for certain items, such as golf shoes) and school supplies are exempt from all sales tax (state and local) on this one weekend only. There has also been talks of a tax free weekend in December to help with the Holiday shopping season.

Utah

Utah has a 4.70% state sales tax. Additionally, local taxing authorities can impose their own sales tax. Currently the majority of Utah's aggregate sales taxes are in the range of 5.95% - 8.35%. Utah has a 16.350% sales tax on rental cars in Salt Lake City.[110] The sales tax on food and food ingredients is 3.0% statewide. This includes the state rate of 1.75%, local option rate of 1.0% and county option rate of .25%.

Vermont

Vermont has a 6% sales tax. Groceries, clothing, prescription and non-prescription drugs are exempt. A separate 9% rooms tax is applied to rentals of hotel and meeting rooms . A 9% meals tax is charged on sales of prepared food and restaurant meals; sales of alcoholic beverages are taxed at 10%. Cities and towns may collect 1% as a local option, effectively establishing a 7% sales tax, a 10% rooms tax, a 10% meals tax on prepared food, and an 11% meals tax on alcoholic beverages. All local option taxes are charged in Dover, Killington, Manchester, Middlebury, Rutland Town, South Burlington, Stratton, Williston, Winhall, and Wilmington. The towns of Brattleboro and Stowe charge the local option rooms and meals taxes, but do not charge the local option sales tax. The cities of Burlington and Rutland do not charge the local option rooms or meals taxes. They are authorized by their respective charters to levy their own taxes on meals, lodging, and entertainment. Burlington does collect the local option sales tax.[111]

Motor vehicle sales are exempt from the general sales tax, but are subject to an equivalent 6% purchase or use tax. Short term auto rentals are taxed at 9%.[112] Gasoline is taxed at 20 cents per gallon, plus an amount equal to 2% of the average state-wide retail price. A Motor Fuel Transportation Infrastructure Assessment fee is also added to the gas tax rate.[113]

Retail sales of "spirituous liquors" have been subject to the 6% general sales tax since July 1, 2009.[114] Prices set by the Vermont Liquor Control Board include the state's 25% gross receipts tax on the sale of liquor and fortified wines, while beer and wine prices reflect the 55 cent per gallon excise tax paid by bottlers or wholesalers.[115]

Virginia

Virginia has a sales tax rate of 5.30% (4.3% state tax and 1% local tax). An additional 0.7% state tax is applied in the Northern Virginia region (Cities of Alexandria, Fairfax, Falls Church, Manassas, and Manassas Park; and Counties of Arlington, Fairfax, Loudoun, and Prince William) and the Hampton Roads region (Cities of Chesapeake, Franklin, Hampton, Newport News, Norfolk, Poquoson, Portsmoth, Suffolk, Virginia Beach, and Williamsburg; and Counties of Isle of Wight, James City, Southampton, and York). Consumers are taxed on every 'eligible food item.' For example, fresh local produce sold at farmers markets and grocery stores, or basic, unprepared cold grocery foods, are taxed 2.5% (1.5% state tax and 1% local tax).[116] Cities and counties may also charge an additional "Food and Beverage Tax" on restaurant meals, up to an additional 4% in counties and 6.5% in cities.[117] Virginia also has a tax on alcohol of 11.5%.

Virginia's use tax also applies at the same rate for out of state purchases (food 2.5%, non-food 5.3% to 6%) exceeding $100 per year "from mail order catalogs".[118] Various exemptions include prescription and non-prescription medicine, gasoline[*citation needed*], and postage stamps, or the labor portion of vehicle repair[*citation needed*]. "Cost price" does not include separately stated "shipping" charges but it does include a separate "handling" charge or "shipping and handling" charges if listed as a combined item on the sales invoice.

However, unlike Maryland and West Virginia consumer use tax forms, the Virginia CU-7 Consumer Use Tax Form does not recognize that it is possible to be *under-taxed* in another state and so only addresses *untaxed* items. Unlike Maryland's quarterly filing, Virginia's CU-7 is due annually between January 1 and May 1 or can be filed optionally instead with Schedule A with Form 760, or Schedule NPY with Form 760PY. As with all states, Virginia has penalties and interest for non-filing, but Virginia's use tax is no more practically enforceable than that of any other state.

Washington

Washington has a 6.5% statewide sales tax. Local rates vary based on an individual's location at the point of purchase. In addition, due to the large number of Native American sovereign nations located within the state, sales-tax rates, if any, can vary based on state treaties with each nation.

As of December 2, 2010, sales taxes cannot be applied to unprepared food items and prescription medications. Prepared food, over-the-counter medications, and medical marijuana are not exempt from sales tax.

The sale or lease of motor vehicles for use on the road incurs an additional 0.3% tax, rental of a car for less than 30 days has an additional state/local tax of 8.9%.[119] When renting a car for less than 30 days in Seattle, the total sales tax is 18.6%. When purchasing an automobile, if one trades in a car, the state subtracts the price of the trade when calculating the sales tax to be paid on the automobile (e.g., purchasing a $40,000 car, and trading a $10,000 car, a person would be taxed on the difference of $30,000 only, not the full amount of the new vehicle).

When staying at a hotel (60+ rooms capacity) in Seattle, the sales tax is 15.6%. Residents of Canada and U.S. states or possessions (only U.S. and Canadian locations having a sales tax of less than 3%, e.g., Oregon, Alaska, and Alberta) are exempt from sales tax on purchases of tangible personal property for use outside the state. Stores at the border will inquire about residency, and exempt qualified purchasers from the tax.

A seller of a house or real estate pays excise taxes on the full sale price. The amount varies by county. In King and Snohomish counties, it is up to 1.78%. For example, selling a house for $500,000 will incur a tax of $8,900.

Residents of Washington, who purchase goods for use in Washington, must pay a use tax in lieu of a sales tax, if any one of four conditions is true: If a Washington resident purchases goods and certain services in other states that do not charge a sales tax or charge a sales tax rate less than the sales tax rate in Washington, or if an out-of-state seller does not collect Washington sales tax, the resident must pay a use tax on all goods that will be used in Washington. Use tax must also be paid if a Washington resident purchases goods from a seller who is not authorized to collect sales tax or if personal property is acquired with the purchase of real property. Washington state does not typically pursue use tax collection for most purchases. However, in 2005, the Washington State Department of Revenue began to make a concerted effort to collect use tax on artworks acquired in other states.

The lowest combined state, county, and municipality sales tax rate in Washington is 7% in most of Klickitat and Skamania counties, while the highest combined sales tax in Washington is the 10% tax on prepared food and beverages in King County.

The highest sales tax in Washington is on liquor and is 20.5%. (This does not include the additional WA Liquor Tax.)

April 1, 2008 saw tax increases in King County (+.001), Kittitas County (+.003), Mason County (+.001), and the city of Union Gap (+.002).[120]

On July 1, 2008, Washington stopped charging an origin-based sales tax, and started charging a destination-based sales tax. This change only applies to transactions beginning and ending within state lines and does not apply to other states. Additionally, Washington started collecting taxes from online retailers that have voluntarily agreed to start collecting the sales tax in return for not being sued for back taxes.

The city of Seattle charges a 10% tax on charges for parking garages to go toward mass transit. Total tax at parking garages is now 19.5%. This rate was set to go up to 12.5% on 1 January 2011, making the parking tax total 22%.

On November 4, 2008, voters in King County (Seattle) approved a 0.5% increase in the sales tax. Taxes within the city were increased to 9.5% on retail purchases. This increase was supposed to be effective January 1, 2009, but was pushed back until April 2009. (For the first quarter of 2009, the tax rate in Seattle was 9%.)

Midterm elections that occurred on November 2, 2010, saw sweeping voter-enacted tax changes with the passage of Initiative 1107. The initiative repealed all taxes levied on candy, gum, snack foods, carbonated beverages, and bottled water that had been imposed by the state legislature on June 1, 2010. Initiative 1107 also lowered the B&O tax on food products that contain meat. The Department of Revenue stated that all tax rollbacks were to take effect

on December 2, 2010, directing retailers to stop collecting the sales tax at that time. The language of the initiative did not order the state to refund any tax that had been previously collected.

West Virginia

West Virginia has the distinction of being the first US state to enact a sales tax.[121] It currently stands at 6%. Effective January 1, 2006, the sales tax on food was lowered to 5%, and on July 1, 2007, it was lowered further to 4%. The sales tax on food was again lowered to 3% on July 1, 2008, to 2% on January 1, 2012, and then to 1% on July 1, 2012, and was finally eliminated on July 1, 2013.[122] However, the reduced rate of tax does not apply to sales, purchases and uses by consumers of prepared food. Prescription drugs are not subject to sales tax. Credit is allowed for sales or use taxes paid to another state with respect to the purchase.

An individual who titles a motor vehicle with the West Virginia Division of Motor Vehicles must pay a $10 title fee and a 5 percent title privilege tax (rather than the 6 percent sales tax). For vehicles purchased new by West Virginia residents, the measure of this tax is the net sales price of the vehicle. For used vehicles, and for vehicles previously titled in other states, the tax is measured by the National Automobile Dealers Association book value of the vehicle at the time of registration. No credit is issued for any taxes paid to another state. Trailers, motorboats, all-terrain vehicles and snowmobiles are also subject to this tax. As of June 7, 2007, new residents of West Virginia no longer have to pay the 5 percent title privilege tax on vehicles, as long as the vehicles were validly titled to the same owner outside the state.

West Virginia is a member of the Streamlined Sales Tax Project. The streamlined sales tax agreement has effected significant in-state tax changes. West Virginia's Energy Star Sales Tax Free Holiday was inspired by similar holidays in other states. The reduction of the food tax was similarly influenced. West Virginia adopted the definition of food found in the Streamlined Sales Tax Agreement in deciding what food is subject to preferential tax rates.

Wisconsin

Wisconsin has a 5% state sales tax, with 62 of the 72 counties charging an extra 0.5% "County Tax".

The state instituted a 0.1% sales tax in five counties (Milwaukee, Ozaukee, Racine, Washington, Waukesha), effective January 1, 1996, to cover the cost of building Miller Park in Milwaukee. The tax was originally scheduled to be retired in 2014; however, it is now not expected to collect enough taxes to cover the entire cost until 2017.[123] There had been talk of extending the tax to pay for a new arena for the Milwaukee Bucks, but the plan failed to gain any momentum.[124]

Brown County collects a 0.5% tax for all purchases which funded the remaining cost of a $295 million renovation of Lambeau Field, as well as a payment to the Green Bay Packers to cover part of the cost of operating and maintaining the stadium through the year 2031. The necessary funds are expected to all be collected by about 2015, when the tax is currently scheduled to end.[125]

The municipalities of Lake Delton, Wisconsin Dells, Bayfield, Stockholm (passed by legislature in 2013), and Eagle River have also been authorized to adopt an additional 1% sales tax, due to their status as popular tourist destinations.[126]

Milwaukee County also has an additional local exposition tax on hotel rooms, car rentals, and food and beverage, which funds the Wisconsin Center Tax District.[127]

In all cases, prescriptions, most foods (with the exception of candy, dietary supplements, soft drinks, and prepared foods), and newspapers, among other items, are exempt from sales tax; however, over-the-counter medications, and certain types of repair and installation services are not tax exempt.[128]

Wyoming

Wyoming has a 4% state sales tax, with counties adding up to an additional 3%, resulting in a maximum rate of 7%. In addition, resort district areas have the option to impose an additional 3% tax. Exemptions include food which is not designed to be consumed on-premises and sales of agricultural equipment. Unlike most states, in Wyoming labor falls under the jurisdiction of sales tax. All labor on both tangible and intangible property are taxable, however labor which permanently modifies real property is exempt.

Internet transactions

In May 2013, the Senate passed the Marketplace Fairness Act, which would allow states to collect sales taxes for purchases made online. Now before the House of Representatives, the legislation would give States the tools to collect sales taxes on cross-State sales transactions. The bill received support from retailers including Walmart and Amazon.com, who have claimed that it is unfair not to require online merchants to collect sales taxes. Groups like the National Retail Federation and the Retail Industry Leaders Association have said that requiring online vendors to collect sales taxes will help make brick and mortar retailers more competitive. However, U.S. House Speaker John Boehner stated that it would be difficult to implement such a system due to varying tax codes in different states. The National Taxpayers Union (NTU) spoke out against the bill, along with The Heritage Foundation, which indicated that it would harm Internet commerce and small businesses. Online retailer Ebay believes it will hurt some of its sellers and lobbied Congress to exempt businesses that have less than $10 million in out-of-state sales or fewer than 50 employees.

Value added tax

There is no value added tax in the United States. There have been proposals to replace some Federal taxes with a value added tax.

History

"The use of sales taxes by U.S. states dates back at least to the Pennsylvania mercantile license tax that was initially introduced in 1821, though this and other early taxes were not broad-based. Buehler attributes development of modern state sales taxes to the depression era. He credits Kentucky with the first tax levied exclusively on retailers. The initial tax, passed in 1930, was progressive, but was replaced in 1934 with a 3 percent flat rate tax and then was eliminated in 1936. The current Kentucky sales tax was adopted in 1960. Commerce ClearingHouse credits Mississippi with the first sales tax, in 1930. Forty-five states and the District of Columbia currently impose sales taxes (see Table 1). Twenty-four of the states first levied the tax during the 1930s, six in the 1940s, five in the 1950s, and eleven in the 1960s. Vermont was the most recent state to impose a sales tax, in 1969. Alaska, Delaware, Montana, New Hampshire, and Oregon do not levy general sales taxes."

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[3] See, *e.g.*, Wyoming Vendor Manual (http://revenue.state.wy.us/PortalVBVS/uploads/Vendor Manual.pdf), page 4.

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[5] For tables of information on state taxes, see, e.g., 2009 State Tax Handbook, CCH, ISBN 978-0-8080-1921-3, or later editions, or All States Handbook, 2010 Edition, RIA Thomson, ISBN 978-0-7811-0415-9 or later editions, or Hellerstein, 12.02. Also see, *e.g.* New York Sales and Use Tax (http://www.tax.ny.gov/bus/st/stidx.htm), which states, New York "Tax Law exempts purchases for resale; most sales to or by

the federal and New York State governments, charitable organizations, and certain other exempt organizations; sales of most food for home consumption; and sales of prescription and nonprescription medicines."

[6] Hellerstein, page 691; Hellerstein, 14.01.

[7] See, *e.g.*, Louisiana Publication 20165 (http://revenue.louisiana.gov/forms/publications/20165(6_10).pdf), quoting Louisiana R.S. 47:301(10) as providing that sales tax is due when the operator of a vending machine purchases the goods, not when they are resold through vending machines.

[8] Sales of food in restaurants are generally taxable, while sales in grocery stores often are not. For example, Texas Publication 96-280 (http://www.window.state.tx.us/taxinfo/taxpubs/tx96_280.pdf), "Grocery and Convenience Stores: Taxable and Nontaxable Sales", differentiates sales of individual sized packages of many foods. It states, "If your store has a dining area, collect sales tax on individual-sized packages of such items as chips … If your store doesn't have eating facilities, do not collect sales tax on chips …" Many other states have such distinctions, but details vary widely.

[9] See Texas publication 94-124 (http://www.window.state.tx.us/taxinfo/taxpubs/tx94_124.html), "Manufacturing Exemptions" for numerous such differences.

[10] Hellerstein 13.10[2].

[11] Texas and California courts have rejected exemptions for sales by religious organizations of their publications on the grounds that such exemption constituted prohibited establishment of religion.

[12] Tennessee Supreme Court, *Broadacre Dairies, Inc. v. Evans*, 246 S.W.2d 78 (1952).

[13] Wyoming Vendor Manual, page 5.

[14] See, *e.g.*, Texas Publication 96-280 for reduction of sales price of groceries for coupons.

[15] See, *e.g.*, Louisiana partial exemption for tangible personal property that is not an ingredient but is consumed in a manufacturing process (such as conveyor belts or motor oils), in Louisiana Publication R-1002 (http://revenue.louisiana.gov/forms/publications/R-1002(10_09).pdf).

[16] States that tax only some food must distinguish between taxable and nontaxable goods, a distinction that is often difficult to make. Distinctions between food that may be consumed on premises or off premises, such as prepackaged foods sold in convenience stores, may be particularly difficult. See Hellerstein, 13.09[5]. The Ohio Constitution provides in Article XII Section 3 that “no excise tax shall be levied or collected upon the sale or purchase of food for human consumption off the premises where sold.” This has led to Ohio Supreme Court cases differentiating sales at mall food courts (taxable) from sales at stadiums (exempt).

[17] See, *e.g.*, New York Publication 880 (http://www.tax.ny.gov/pdf/publications/sales/pub880.pdf) for an incomplete list of taxable and exempt foods and beverages.

[18] For example, in defining the term "baked goods," certain of which are exempt from sales tax, the Texas Administrative Code Section 3.293(a) (http://info.sos.state.tx.us/pls/pub/readtac$ext.TacPage?sl=R&app=9&p_dir=&p_rloc=&p_tloc=&p_ploc=&pg=1&p_tac=&ti=34&pt=1&ch=3&rl=293) states, "(1) Bakery items--Baked goods typically made by bakeries including bread, rolls, buns, biscuits, bagels, croissants, pastries, doughnuts, Danishes, cakes, tortes, pies, tarts, muffins, bars such as lemon bars, cookies, large pretzels, and tortillas. The term does not include candy; snack items including chips, small pretzels, or crackers; sandwiches; tacos; or pizzas."

[19] Hellerstein, page 701.

[20] See, *e.g.*, examples in New York sales tax regulation 527.4.

[21] Few states, however, tax customized software or computer programming services. Contrast legal treatment as a license of software to the Federal income tax treatment as sale of a copyrighted item. See 26 CFR 1.861-17. Some states have adopted similar views for sales tax treating sales of software the same as sales of books.

[22] See *Navistar International Transportation Corporation v. State Board of Equalization*, 8 Cal.4th 868 (Supreme Court of California, 1994), cited in Hellerstein, page 710. The Court held that Navistar's sale of intellectual property (turbine engine designs) as part of a sale of a whole division was a taxable sale. Hellerstein notes, "There are literally hundreds of cases" in which state courts have ruled on characterization of sales involving intangibles.

[23] Hellerstein 14.02. This is done in the definition of "retail" sale in some states, such as New York, and by explicitly excluding purchases for resale in others, such as California.

[24] California and Georgia courts have reached opposite conclusions on this issue with respect to vehicles used as demonstration cars by dealers.

[25] Contrast Missouri's' nontaxability with Louisiana's and Washington's taxability of purchases of cellular phones to be given at low or no cost as part of cellular phone service agreements.

[26] In *Sta-Ru Corp. v. Mahin*, 64 Ill. 2d 330, 356 NE2d 67 (1976), Illinois taxed purchases by restaurants of fast food containers, even though the food sold in the containers was later taxed. New York's courts have held similarly. Arizona courts have found that containers, napkins, etc., should be considered part of the cost of the meal, so that the restaurant paid no sales tax on their purchase. Arkansas and Nebraska courts have held that reactive chemicals and hot dog casings are not themselves in the final product and therefore their purchase by manufacturers is taxable. By contrast, New York and Washington courts have held that purchases of chemicals and fuels used in certain manufacturing exempt as ingredients. See Hellerstein 14.03[1] and 14.03[2].

[27] Hellerstein, 13.09.

[28] But the ability of states to collect such tax may be limited. See, *e.g.*, 41 USC 40116 (http://www.law.cornell.edu/uscode/html/uscode49/usc_sec_49_00040116----000-.html).

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[30] Hellerstein 12.08.

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[33] Quill Corp. v. North Dakota and National Bellas Hess v. Illinois both prohibit states from imposing a sales and use tax collection obligation on out of state sellers with no nexus in the state.
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External links

- State Sales, Gasoline, Cigarette, and Alcohol Tax Rates by State (http://www.taxfoundation.org/taxdata/show/245.html), Tax Foundation
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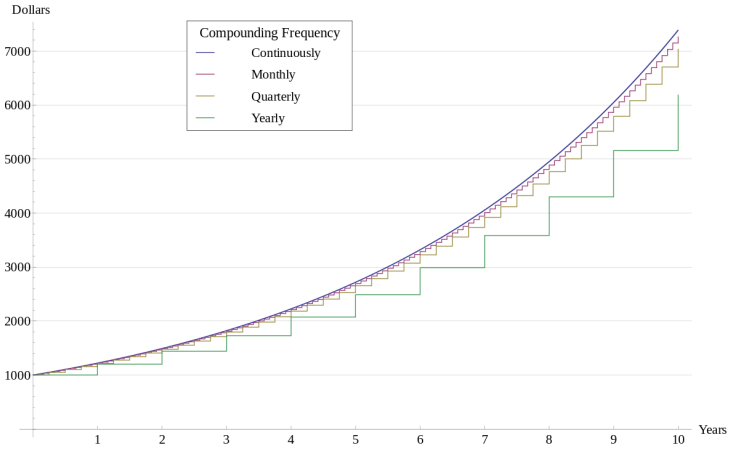
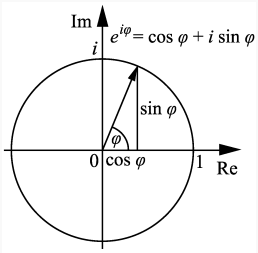
Compound interest



The effect of earning 20% annual interest on an initial $1,000 investment at various compounding frequencies

Part of a series of articles on the
mathematical constant *e*

Properties
• Natural logarithm • Exponential function
Applications
• compound interest • Euler's identity • Euler's formula • half-lives • exponential growth and decay
Defining *e*
• proof that *e* is irrational • representations of *e* • Lindemann–Weierstrass theorem
People
• John Napier • Leonhard Euler
Related topics
• Schanuel's conjecture
• v • t • e [1]

Compound interest arises when interest is added to the principal of a deposit or loan, so that, from that moment on, the interest that has been added also earns interest. This addition of interest to the principal is called *compounding*. A bank account, for example, may have its interest compounded every year: in this case, an account with $1000 initial principal and 20% interest per year would have a balance of $1200 at the end of the first year, $1440 at the end of the second year, and so on.

In order to define an interest rate fully, and enable one to compare it with other interest rates, the interest rate *and* the compounding frequency must be disclosed. Since most people prefer to think of rates as a yearly percentage, many governments require financial institutions to disclose the equivalent yearly compounded interest rate on deposits or advances. For instance, the yearly rate for a loan with 1% interest per month is approximately 12.68% per annum ($1.01^{12} - 1$). This equivalent yearly rate may be referred to as *annual percentage rate* (APR), *annual equivalent rate* (AER), *effective interest rate*, *effective annual rate*, and by other terms. When a fee is charged up front to obtain a loan, APR usually counts that cost as well as the compound interest in converting to the equivalent rate. These government requirements assist consumers in comparing the actual costs of borrowing more easily.

For any given interest rate and compounding frequency, an "equivalent" rate for any different compounding frequency exists.

Compound interest may be contrasted with simple interest, where interest is not added to the principal (there is no compounding). Compound interest is standard in finance and economics, and simple interest is used infrequently (although certain financial products may contain elements of simple interest).

Terminology

The effect of compounding depends on the frequency with which interest is compounded and the periodic interest rate which is applied. Therefore, in order to define accurately the amount to be paid under a legal contract with interest, the frequency of compounding (yearly, half-yearly, quarterly, monthly, daily, etc.) *and* the interest rate must be specified. Different conventions may be used from country to country, but in finance and economics the following usages are common:

The *periodic rate* is the amount of interest that is charged (and subsequently compounded) for each period, divided by the amount of the principal. The periodic rate is used primarily for calculations, and is rarely used for comparison.

The *nominal annual rate* or *nominal interest rate* is defined as the periodic rate multiplied by the number of compounding periods per year. For example, a monthly rate of 1% is equivalent to an annual nominal interest rate of 12%.

The *effective annual rate* is the total accumulated interest that would be payable up to the end of one year, divided by the principal.

Economists generally prefer to use effective annual rates to allow for comparability. In finance and commerce, the nominal annual rate may however be the one quoted instead. When quoted together with the compounding frequency, a loan with a given nominal annual rate is fully specified (the amount of interest for a given loan scenario can be precisely determined), but the nominal rate cannot be directly compared with that of loans that have a different compounding frequency.

Loans and finance may have other "non-interest" charges, and the terms above do not attempt to capture these differences. Other terms such as annual percentage rate and annual percentage yield may have specific legal definitions and may or may not be comparable, depending on the jurisdiction.

The use of the terms above (and other similar terms) may be inconsistent, and vary according to local custom or marketing demands, for simplicity or for other reasons.

Exceptions

- US and Canadian T-Bills (short term Government debt) have a different convention. Their interest is calculated as $(100 - P)/Pbnm$,Wikipedia:Please clarify where P is the price paid. Instead of normalizing it to a year, the interest is prorated by the number of days t: $(365/t)\times 100$. (See day count convention).
- The interest on corporate bonds and government bonds is usually payable twice yearly. The amount of interest paid (each six months) is the disclosed interest rate divided by two, multiplied by the principal. The yearly compounded rate is higher than the disclosed rate.
- Canadian mortgage loans are generally compounded semi-annually with monthly (or more frequent) payments.[2]
- U.S. mortgages use an amortizing loan, not compound interest. With these loans, an amortization schedule is used to determine how to apply payments toward principal and interest. Interest generated on these loans is not added to the principal, but rather is paid off monthly as the payments are applied.
- It is sometimes mathematically simpler, e.g. in the valuation of derivatives, to use *continuous compounding*, which is the limit as the compounding period approaches zero. Continuous compounding in pricing these instruments is a natural consequence of Itō calculus, where financial derivatives are valued at ever increasing frequency, until the limit is approached and the derivative is valued in continuous time.

Mathematics of interest rates

Simplified calculation

Formulae are presented in greater detail at time value of money.

In the formulae below, *i* is the effective interest rate per period. *FV* and *PV* represent the future and present value of a sum. *n* represents the number of periods.

These are the most basic formulas:

$$FV = PV(1+i)^n$$

The above calculates the future value (*FV*) of an investment whose present value is *PV* accruing interest at a fixed interest rate (*i*) for *n* periods.

$$PV = \frac{FV}{(1+i)^n}$$

The above calculates what present value (*PV*) would be needed to produce a specified future value (*FV*) if interest accrues at the rate *i* for *n* periods.

$$i = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1$$

The above calculates the compound interest rate achieved if an initial investment of *PV* returns a value of *FV* after *n* accrual periods.

$$n = \frac{\log(FV) - \log(PV)}{\log(1+i)}$$

The above formula calculates the number of periods required to get *FV* given the *PV* and the interest rate (*i*). The log function can be in any base, e.g. natural log (ln), as long as consistent bases are used throughout the calculation.

Compound Interest

A formula for calculating annual compound interest as follows:

$$S = P\left(1 + \frac{j}{m}\right)^{mt}$$

where

- *S* = value after *t* periods
- *P* = principal amount (initial investment)
- *j* = annual nominal interest rate (not reflecting the compounding)
- *m* = number of times the interest is compounded per year
- *t* = number of years the money is borrowed for

As an example, suppose an amount of 1500.00 is deposited in a bank paying an annual interest rate of 4.3%, compounded quarterly. Then the balance after 6 years is found by using the formula above, with *P* = 1500, *j* = 0.043 (4.3%), *m* = 4, and *t* = 6:

$$S = 1500\left(1 + \frac{0.043}{4}\right)^{4\times 6} = 1938.84$$

So, the balance after 6 years is approximately 1938.84. The amount of interest received can be calculated by subtracting the principal from this amount.

Periodic compounding

The amount function for compound interest is a power law function in terms of time.

$$A(t) = A_0 \left(1 + \frac{r}{n}\right)^{\lfloor nt \rfloor}$$

- t = Total time in years
- n = Number of compounding periods per year (note that the total number of compounding periods is $n \cdot t$)
- r = Nominal annual interest rate expressed as a decimal. e.g.: 6% = 0.06
- $\lfloor nt \rfloor$ means that *nt* is rounded down to the nearest integer.

As *n*, the number of compounding periods per year, increases without limit, we have the case known as *continuous compounding*, in which case the effective annual rate approaches an upper limit of $e^r - 1$.

Since the principal $A(0)$ is simply a coefficient, it is often dropped for simplicity, and the resulting accumulation function is used in interest theory instead. Accumulation functions for simple and compound interest are listed below:

$$a(t) = 1 + tr$$

$$a(t) = \left(1 + \frac{r}{n}\right)^{nt}$$

Note: $A(t)$ is the amount function and $a(t)$ is the accumulation function.

Continuous compounding

Continuous compounding can be thought of as making the compounding period infinitesimally small, achieved by taking the limit as *n* goes to infinity. See definitions of the exponential function for the mathematical proof of this limit. The amount after *t* periods of continuous compounding can be expressed in terms of the initial amount A_0 as

$$A(t) = A_0 e^{rt}.$$

Force of interest

In mathematics, the accumulation functions are often expressed in terms of *e*, the base of the natural logarithm. This facilitates the use of calculus methods in manipulation of interest formulae.

For any continuously differentiable accumulation function *a(t)* the force of interest, or more generally the logarithmic or continuously compounded return is a function of time defined as follows: $\delta_t = \frac{a'(t)}{a(t)}$

which is the rate of change with time of the natural logarithm of the accumulation function.

Conversely: $a(n) = e^{\int_0^n \delta_t \, dt}$,(since $a(0) = 1$)

When the above formula is written in differential equation format, the force of interest is simply the coefficient of amount of change: $da(t) = \delta_t a(t)\, dt$

For compound interest with a constant annual interest rate *r*, the force of interest is a constant, and the accumulation function of compounding interest in terms of force of interest is a simple power of e: $\delta = \ln(1 + r)$or $a(t) = e^{t\delta}$

The force of interest is less than the annual effective interest rate, but more than the annual effective discount rate. It is the reciprocal of the e-folding time. See also notation of interest rates.

A way of modeling the force of inflation is with Stoodley's formula: $\delta_t = p + \frac{s}{1 + rse^{st}}$where p, r and s are estimated.

Compounding basis

To convert an interest rate from one compounding basis to another compounding basis, the following formula applies:

$$r_2 = \left[\left(1 + \frac{r_1}{n_1}\right)^{\frac{n_1}{n_2}} - 1\right] n_2$$

where r_1 is the stated interest rate with compounding frequency n_1 and r_2 is the stated interest rate with compounding frequency n_2.

When interest is continuously compounded:

$$R = n \ln\left(1 + r/n\right)$$

where *R* is the interest rate on a continuous compounding basis and *r* is the stated interest rate with a compounding frequency *n*.

Mathematics of interest rate on loans

Monthly amortized loan or mortgage payments

The interest on loans and mortgages that are amortized—that is, have a smooth monthly payment until the loan has been paid off—is often compounded monthly. The formula for payments is found from the following argument.

Exact formula for monthly payment

An exact formula for the monthly payment is

$$P = \frac{Li}{1 - \frac{1}{(1+i)^n}}$$

or equivalently

$$P = \frac{Li}{1 - e^{-n\ln(1+i)}}$$

This can be derived by considering how much is left to be repaid after each month. After the first month $L_1 = (1+i)L - P$ is left, i.e. the initial amount has increased less the payment. If the whole loan was repaid after a month then $L_1 = 0$ so $L = \frac{P}{1+i}$ After the second month $L_2 = (1+i)L_1 - P$ is left, that is $L_2 = (1+i)((1+i)L - P) - P$. If the whole loan was repaid after two months $L_2 = 0$ this gives the equation $L = \frac{P}{1+i} + \frac{P}{(1+i)^2}$. This equation generalises for a term of n months, $L = P\sum_{j=1}^{n} \frac{1}{(1+i)^j}$. This is a geometric series which has the sum

$$L = \frac{P}{i}\left(1 - \frac{1}{(1+i)^n}\right)$$

which can be rearranged to give

$$P = \frac{Li}{1 - \frac{1}{(1+i)^n}} = \frac{Li}{1 - e^{-n\ln(1+i)}}$$

This formula for the monthly payment on a U.S. mortgage is exact and is what banks use.

In Excel, the function **PMT()** function is used. The syntax for the PMT function is:

= - ***PMT(interest_rate, number_payments, PV, [FV],[Type])***

See http://office.microsoft.com/en-au/excel-help/pmt-HP005209215.aspx for more details.

For example, for interest rate of 6% (0.06/12 p.m.), 25 years * 12 p.a., PV of $150,000, FV of 0, type of 0 gives:

= - ***PMT(0.06/12, 25 * 12, 150000, 0, 0)***

= $ 966.45 p.m.

Approximate formula for monthly payment

A formula that is accurate to within a few percent can be found by noting that for typical U.S. note rates ($I < 8\%$ and terms T=10–30 years), the monthly note rate is small compared to 1: $i << 1$ so that the $ln(1+i) \approx i$ which yields a simplification so that $P \approx \frac{Li}{1-e^{-ni}} = \frac{L}{n}\frac{ni}{1-e^{-ni}}$

which suggests defining auxiliary variables

$Y \equiv ni = TI$

$P_0 \equiv \frac{L}{n}.$

P_0 is the monthly payment required for a zero interest loan paid off in n installments. In terms of these variables the approximation can be written

$P \approx P_0 \frac{Y}{1-e^{-Y}}$

The function $f(Y) \equiv \frac{Y}{1-e^{-Y}} - \frac{Y}{2}$ is even: $f(Y) = f(-Y)$ implying that it can be expanded in even powers of Y.

It follows immediately that $\frac{Y}{1-e^{-Y}}$ can be expanded in even powers of Y plus the single term: $Y/2$

It will prove convenient then to define

$X = \frac{1}{2}Y = \frac{1}{2}IT$

so that $P \approx P_0 \frac{2X}{1-e^{-2X}}$ which can be expanded: $P \approx P_0 \left(1 + X + \frac{X^2}{3} - \frac{1}{45}X^4 + ...\right)$

where the ellipses indicate terms that are higher order in even powers of X . The expansion

$P \approx P_0 \left(1 + X + \frac{X^2}{3}\right)$

is valid to better than 1% provided $X \leq 1$.

Example of mortgage payment

For a mortgage with a term of 30 years and a note rate of 4.5% we find:

$T = 30$

$I = .045$ ⌜⌜⌜⌜≃

</math>

so that

$P \approx P_0 \left(1 + X + \frac{1}{3}X^2\right) = \$333.33(1 + .675 + .675^2/3) = \$608.96$

The exact payment amount is $P = \$608.02$ so the approximation is an overestimate of about a sixth of a percent.

Example of compound interest

Suppose that one cent had been invested in a bank 2012 years ago at a 5% interest rate maintained to the present. After the first year the capital would be worth 5% more than one cent, or 1.05 cents. In the second year the interest earned would be 5% times 1.05 cents, giving the amount of 1.05×1.05. After three years it would have grown to $(1.05)^3$. After 2012 years the original one cent contribution would have grown to $(1.05)^{2012}$cents, or $4.29 \cdot 10^{42}$cents (more accurately, a vast 4,294,076,020,321,072,693, 082,856,331,311,709,813,735,424 of them).

History

Compound interest was once regarded as the worst kind of usury, and was severely condemned by Roman law, as well as the common laws of many other countries.

In one passage, the Bible addresses the charging of interest in the following manner:

> Take thou no usury of him, or increase: but fear thy God; that thy brother may live with thee. Thou shalt not give him thy money upon usury, nor lend him thy victuals for increase.
>
> — Leviticus 25:36-37 [3]

The Qur'an explicitly mentions compound interest as a great sin. Usury (oppressive interest), known in Arabic as "riba", is considered wrong:

> O ye who believe! Devour not usury, doubling and quadrupling (the sum lent). Observe your duty to Allah, that ye may be successful.
>
> — Quran 3:130 [4]

Richard Witt's book *Arithmeticall Questions*, published in 1613, was a landmark in the history of compound interest. It was wholly devoted to the subject (previously called **anatocism**), whereas previous writers had usually treated compound interest briefly in just one chapter in a mathematical textbook. Witt's book gave tables based on 10% (the then maximum rate of interest allowable on loans) and on other rates for different purposes, such as the valuation of property leases. Witt was a London mathematical practitioner and his book is notable for its clarity of expression, depth of insight and accuracy of calculation, with 124 worked examples.

References

[1] http://en.wikipedia.org/w/index.php?title=Template:E_(mathematical_constant)&action=edit

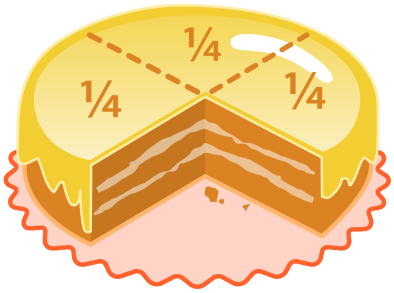
[2] http://laws.justice.gc.ca/en/showdoc/cs/I-15/bo-ga:s_6//en#anchorbo-ga:s_6 Interest Act (Canada), *Department of Justice*. The Interest Act specifies that interest is not recoverable unless the mortgage loan contains a statement showing the rate of interest chargeable, "calculated yearly or half-yearly, not in advance." In practice, banks use the half-yearly rate.

[3] http://tools.wmflabs.org/bibleversefinder/?book=Leviticus&verse=25:36-37&src=!

[4] http://www.usc.edu/org/cmje/religious-texts/quran/verses/003-qmt.php#003.130

Fraction (mathematics)

A **fraction** (from Latin: **fractus**, "broken") represents a part of a whole or, more generally, any number of equal parts. When spoken in everyday English, a fraction describes how many parts of a certain size there are, for example, one-half, eight-fifths, three-quarters. A *common*, *vulgar*, or *simple* fraction (examples: $\frac{1}{2}$and 17/3) consists of an integer **numerator**, displayed above a line (or before a slash), and a non-zero integer **denominator**, displayed below (or after) that line. Numerators and denominators are also used in fractions that are not *common*, including compound fractions, complex fractions, and mixed numerals.



A cake with one fourth (a quarter) removed. The remaining three fourths are shown. Dotted lines indicate where the cake may be cut in order to divide it into equal parts. Each fourth of the cake is denoted by the fraction ¼.

The numerator represents a number of equal parts, and the denominator, which cannot be zero, indicates how many of those parts make up a unit or a whole. For example, in the fraction 3/4, the numerator, 3, tells us that the fraction represents 3 equal parts, and the denominator, 4, tells us that 4 parts make up a whole. The picture to the right illustrates $\frac{3}{4}$or 3/4 of a cake.

Fractional numbers can also be written without using explicit numerators or denominators, by using decimals, percent signs, or negative exponents (as in 0.01, 1%, and 10^{-2} respectively, all of which are equivalent to 1/100). An integer such as the number 7 can be thought of as having an implied denominator of one: 7 equals 7/1.

Other uses for fractions are to represent ratios and to represent division.[1] Thus the fraction 3/4 is also used to represent the ratio 3:4 (the ratio of the part to the whole) and the division 3 ÷ 4 (three divided by four).

In mathematics the set of all numbers which can be expressed in the form a/b, where a and b are integers and b is not zero, is called the set of rational numbers and is represented by the symbol **Q**, which stands for quotient. The test for a number being a rational number is that it can be written in that form (i.e., as a common fraction). However, the word *fraction* is also used to describe mathematical expressions that are not rational numbers, for example algebraic fractions (quotients of algebraic expressions), and expressions that contain irrational numbers, such as √2/2 (see square root of 2) and π/4 (see proof that π is irrational).

Vocabulary

In the examples 2/5 and 7/3, the slanting line is called a solidus or forward slash. In the examples $\frac{2}{5}$and $\frac{7}{3}$, the horizontal line is called a vinculum or, informally, a "fraction bar".

When reading fractions it is customary in English to pronounce the denominator using the corresponding ordinal number, in plural if the numerator is not one, as in "fifths" for fractions with a 5 in the denominator. Thus 3/5 is rendered as *three fifths* and 5/32 as *five thirty-seconds*. This generally applies to whole number denominators greater than 2, though large denominators that are not powers of ten are often rendered using the cardinal number. Thus, 5/123 might be rendered as "five one-hundred-twenty-thirds", but is often "five over one hundred twenty-three". In contrast, because one million is a power of ten, 6/1,000,000 is usually expressed as "six millionths" or "six one-millionths", rather than as "six *over* one million".

The denominators 1, 2, and 4 are special cases. The fraction 3/1 may be spoken of as *three wholes*. The denominator 2 is expressed as *half* (plural *halves*); "$-3/2$" is *minus three-halves* or *negative three-halves*. The fraction 3/4 may be either "three fourths" or "three quarters". Furthermore, since most fractions in prose function as adjectives, the fractional modifier is hyphenated. This is evident in standard prose in which one might write about "every two-tenths of a mile", "the quarter-mile run", or the Three-Fifths Compromise. When the fraction's numerator is 1, then the

word *one* may be omitted, such as "every tenth of a second" or "during the final quarter of the year".

Forms of fractions

Simple, common, or vulgar fractions

A **simple fraction** (also known as a **common fraction** or **vulgar fraction**) is a rational number written as *a*/*b* or $\frac{a}{b}$, where *a* and *b* are both integers. As with other fractions, the denominator (*b*) cannot be zero. Examples include $\frac{1}{2}$, $-\frac{8}{5}$, $\frac{-8}{5}$, $\frac{8}{-5}$, and 3/17. *Simple fractions* can be positive or negative, proper, or improper (see below). Compound fractions, complex fractions, mixed numerals, and decimals (see below) are not *simple fractions*, though, unless irrational, they can be evaluated to a simple fraction.

Proper and improper fractions

Common fractions can be classified as either proper or improper. When the numerator and the denominator are both positive, the fraction is called proper if the numerator is less than the denominator, and improper otherwise.[2] In general, a common fraction is said to be a **proper fraction** if the absolute value of the fraction is strictly less than one—that is, if the fraction is greater than −1 and less than 1.[3][4] It is said to be an **improper fraction**, or sometimes informally **top-heavy fraction**, if the absolute value of the fraction is greater than or equal to 1. Examples of proper fractions are 2/3, -3/4, and 4/9; examples of improper fractions are 9/4, -4/3, and 3/3.

Mixed numbers

A **mixed numeral** (often called a *mixed number*, also called a *mixed fraction*) is the sum of a non-zero integer and a proper fraction. This sum is implied without the use of any visible operator such as "+". For example, in referring to two entire cakes and three quarters of another cake, the whole and fractional parts of the number are written next to each other: $2 + \frac{3}{4} = 2\frac{3}{4}$.

This is not to be confused with the algebra rule of implied multiplication. When two algebraic expressions are written next to each other, the operation of multiplication is said to be "understood". In algebra, $a\frac{b}{c}$ for example is not a mixed number. Instead, multiplication is understood where $a\frac{b}{c} = a \times \frac{b}{c}$.

To avoid confusion, the multiplication is often explicitly expressed. So $a\frac{b}{c}$ may be written as

$a \times \frac{b}{c}$,

$a \cdot \frac{b}{c}$, or

$a\left(\frac{b}{c}\right)$.

An improper fraction is another way to write a whole plus a part. A mixed number can be converted to an improper fraction as follows:

1. Write the mixed number $2\frac{3}{4}$ as a sum $2 + \frac{3}{4}$.
2. Convert the whole number to an improper fraction with the same denominator as the fractional part, $2 = \frac{8}{4}$.
3. Add the fractions. The resulting sum is the improper fraction. In the example, $2\frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}$.

Similarly, an improper fraction can be converted to a mixed number as follows:

1. Divide the numerator by the denominator. In the example, $\frac{11}{4}$, divide 11 by 4. 11 ÷ 4 = 2 with remainder 3.
2. The quotient (without the remainder) becomes the whole number part of the mixed number. The remainder becomes the numerator of the fractional part. In the example, 2 is the whole number part and 3 is the numerator of the fractional part.
3. The new denominator is the same as the denominator of the improper fraction. In the example, they are both 4. Thus $\frac{11}{4} = 2\frac{3}{4}$.

Mixed numbers can also be negative, as in $-2\frac{3}{4}$, which equals $-(2+\frac{3}{4}) = -2-\frac{3}{4}$.

Ratios

A ratio is a relationship between two or more numbers that can be sometimes expressed as a fraction. Typically, a number of items are grouped and compared in a ratio, specifying numerically the relationship between each group. Ratios are expressed as "group 1 to group 2 ... to group *n*". For example, if a car lot had 12 vehicles, of which

- 2 are white,
- 6 are red, and
- 4 are yellow,

then the ratio of red to white to yellow cars is 6 to 2 to 4. The ratio of yellow cars to white cars is 4 to 2 and may be expressed as 4:2 or 2:1.

A ratio is often converted to a fraction when it is expressed as a ratio to the whole. In the above example, the ratio of yellow cars to all the cars on the lot is 4:12 or 1:3. We can convert these ratios to a fraction and say that 4/12 of the cars or 1/3 of the cars in the lot are yellow. Therefore, if a person randomly chose one car on the lot, then there is a one in three chance or probability that it would be yellow.

Reciprocals and the "invisible denominator"

The **reciprocal** of a fraction is another fraction with the numerator and denominator exchanged. The reciprocal of $\frac{3}{7}$, for instance, is $\frac{7}{3}$. The product of a fraction and its reciprocal is 1, hence the reciprocal is the multiplicative inverse of a fraction. Any integer can be written as a fraction with the number one as denominator. For example, 17 can be written as $\frac{17}{1}$, where 1 is sometimes referred to as the *invisible denominator*. Therefore, every fraction or integer except for zero has a reciprocal. The reciprocal of 17 is $\frac{1}{17}$.

Complex fractions

In a **complex fraction**, either the numerator, or the denominator, or both, is a fraction or a mixed number, corresponding to division of fractions. For example, $\frac{\frac{1}{2}}{\frac{1}{3}}$ and $\frac{12\frac{3}{4}}{26}$ are complex fractions. To reduce a complex fraction to a simple fraction, treat the longest fraction line as representing division. For example:

$$\frac{\frac{1}{2}}{\frac{1}{3}} = \tfrac{1}{2} \times \tfrac{3}{1} = \tfrac{3}{2} = 1\tfrac{1}{2}$$

$$\frac{12\frac{3}{4}}{26} = 12\tfrac{3}{4} \cdot \tfrac{1}{26} = \tfrac{12\cdot 4+3}{4} \cdot \tfrac{1}{26} = \tfrac{51}{4} \cdot \tfrac{1}{26} = \tfrac{51}{104}$$

$$\frac{\frac{3}{2}}{5} = \tfrac{3}{2} \times \tfrac{1}{5} = \tfrac{3}{10}$$

$$\frac{8}{\frac{1}{3}} = 8 \times \tfrac{3}{1} = 24.$$

If, in a complex fraction, there is no clear way to tell which fraction lines takes precedence, then the expression is improperly formed, and ambiguous. Thus 5/10/20/40 is a poorly constructed mathematical expression, with multiple possible values.

Compound fractions

A **compound fraction** is a fraction of a fraction, or any number of fractions connected with the word *of*, corresponding to multiplication of fractions. To reduce a compound fraction to a simple fraction, just carry out the multiplication (see the section on multiplication). For example, $\frac{3}{4}$of $\frac{5}{7}$is a compound fraction, corresponding to $\frac{3}{4} \times \frac{5}{7} = \frac{15}{28}$. The terms compound fraction and complex fraction are closely related and sometimes one is used as a synonym for the other.

Decimal fractions and percentages

A **decimal fraction** is a fraction whose denominator is not given explicitly, but is understood to be an integer power of ten. Decimal fractions are commonly expressed using decimal notation in which the implied denominator is determined by the number of digits to the right of a decimal separator, the appearance of which (e.g., a period, a raised period (•), a comma) depends on the locale (for examples, see decimal separator). Thus for 0.75 the numerator is 75 and the implied denominator is 10 to the second power, *viz.* 100, because there are two digits to the right of the decimal separator. In decimal numbers greater than 1 (such as 3.75), the fractional part of the number is expressed by the digits to the right of the decimal (with a value of 0.75 in this case). 3.75 can be written either as an improper fraction, 375/100, or as a mixed number, $3\frac{75}{100}$.

Decimal fractions can also be expressed using scientific notation with negative exponents, such as 6.023×10^{-7}, which represents 0.0000006023. The 10^{-7} represents a denominator of 10^7. Dividing by 10^7 moves the decimal point 7 places to the left.

Decimal fractions with infinitely many digits to the right of the decimal separator represent an infinite series. For example, 1/3 = 0.333... represents the infinite series 3/10 + 3/100 + 3/1000 + ...

Another kind of fraction is the percentage (Latin *per centum* meaning "per hundred", represented by the symbol %), in which the implied denominator is always 100. Thus, 51% means 51/100. Percentages greater than 100 or less than zero are treated in the same way, e.g. 311% equals 311/100, and −27% equals −27/100.

The related concept of *permille* or *parts per thousand* has an implied denominator of 1000, while the more general parts-per notation, as in 75 parts per million, means that the proportion is 75/1,000,000.

Whether common fractions or decimal fractions are used is often a matter of taste and context. Common fractions are used most often when the denominator is relatively small. By mental calculation, it is easier to multiply 16 by 3/16 than to do the same calculation using the fraction's decimal equivalent (0.1875). And it is more accurate to multiply 15 by 1/3, for example, than it is to multiply 15 by any decimal approximation of one third. Monetary values are commonly expressed as decimal fractions, for example $3.75. However, as noted above, in pre-decimal British currency, shillings and pence were often given the form (but not the meaning) of a fraction, as, for example 3/6 (read "three and six") meaning 3 shillings and 6 pence, and having no relationship to the fraction 3/6.

Special cases

- A unit fraction is a vulgar fraction with a numerator of 1, e.g. $\frac{1}{7}$. Unit fractions can also be expressed using negative exponents, as in 2^{-1} which represents 1/2, and 2^{-2} which represents $1/(2^2)$ or 1/4.
- An Egyptian fraction is the sum of distinct positive unit fractions, for example $\frac{1}{2} + \frac{1}{3}$. This definition derives from the fact that the ancient Egyptians expressed all fractions except $\frac{1}{2}$, $\frac{2}{3}$and $\frac{3}{4}$in this manner. Every positive rational number can be expanded as an Egyptian fraction. For example, $\frac{5}{7}$can be written as $\frac{1}{2} + \frac{1}{6} + \frac{1}{21}$. Any positive rational number can be written as a sum of unit fractions in infinitely many ways. Two ways to write $\frac{13}{17}$ are $\frac{1}{2} + \frac{1}{4} + \frac{1}{68}$and $\frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{68}$.
- A dyadic fraction is a vulgar fraction in which the denominator is a power of two, e.g. $\frac{1}{8}$.

Arithmetic with fractions

Like whole numbers, fractions obey the commutative, associative, and distributive laws, and the rule against division by zero.

Equivalent fractions

Multiplying the numerator and denominator of a fraction by the same (non-zero) number results in a fraction that is equivalent to the original fraction. This is true because for any non-zero number n, the fraction $\frac{n}{n} = 1$. Therefore, multiplying by $\frac{n}{n}$ is equivalent to multiplying by one, and any number multiplied by one has the same value as the original number. By way of an example, start with the fraction $\frac{1}{2}$. When the numerator and denominator are both multiplied by 2, the result is $\frac{2}{4}$, which has the same value (0.5) as $\frac{1}{2}$. To picture this visually, imagine cutting a cake into four pieces; two of the pieces together ($\frac{2}{4}$) make up half the cake ($\frac{1}{2}$).

Dividing the numerator and denominator of a fraction by the same non-zero number will also yield an equivalent fraction. This is called reducing or simplifying the fraction. A simple fraction in which the numerator and denominator are coprime (that is, the only positive integer that goes into both the numerator and denominator evenly is 1) is said to be irreducible, in lowest terms, or in simplest terms. For example, $\frac{3}{9}$ is not in lowest terms because both 3 and 9 can be exactly divided by 3. In contrast, $\frac{3}{8}$ *is* in lowest terms—the only positive integer that goes into both 3 and 8 evenly is 1.

Using these rules, we can show that $\frac{5}{10} = \frac{1}{2} = \frac{10}{20} = \frac{50}{100}$.

A common fraction can be reduced to lowest terms by dividing both the numerator and denominator by their greatest common divisor. For example, as the greatest common divisor of 63 and 462 is 21, the fraction $\frac{63}{462}$ can be reduced to lowest terms by dividing the numerator and denominator by 21:

$$\frac{63}{462} = \frac{63 \div 21}{462 \div 21} = \frac{3}{22}$$

The Euclidean algorithm gives a method for finding the greatest common divisor of any two positive integers.

Comparing fractions

Comparing fractions with the same denominator only requires comparing the numerators.

$\frac{3}{4} > \frac{2}{4}$ because 3>2.

If two positive fractions have the same numerator, then the fraction with the smaller denominator is the larger number. When a whole is divided into equal pieces, if fewer equal pieces are needed to make up the whole, then each piece must be larger. When two positive fractions have the same numerator, they represent the same number of parts, but in the fraction with the smaller denominator, the parts are larger.

One way to compare fractions with different numerators and denominators is to find a common denominator. To compare $\frac{a}{b}$ and $\frac{c}{d}$, these are converted to $\frac{ad}{bd}$ and $\frac{bc}{bd}$. Then *bd* is a common denominator and the numerators *ad* and *bc* can be compared.

$\frac{2}{3} ? \frac{1}{2}$ gives $\frac{4}{6} > \frac{3}{6}$

It is not necessary to determine the value of the common denominator to compare fractions. This short cut is known as "cross multiplying" – you can just compare *ad* and *bc*, without computing the denominator.

$\frac{5}{18} ? \frac{4}{17}$

Multiply top and bottom of each fraction by the denominator of the other fraction, to get a common denominator:

$\frac{5 \times 17}{18 \times 17} ? \frac{4 \times 18}{17 \times 18}$

The denominators are now the same, but it is not necessary to calculate their value – only the numerators need to be compared. Since 5×17 (= 85) is greater than 4×18 (= 72), $\frac{5}{18} > \frac{4}{17}$.

Also note that every negative number, including negative fractions, is less than zero, and every positive number, including positive fractions, is greater than zero, so every negative fraction is less than any positive fraction.

Addition

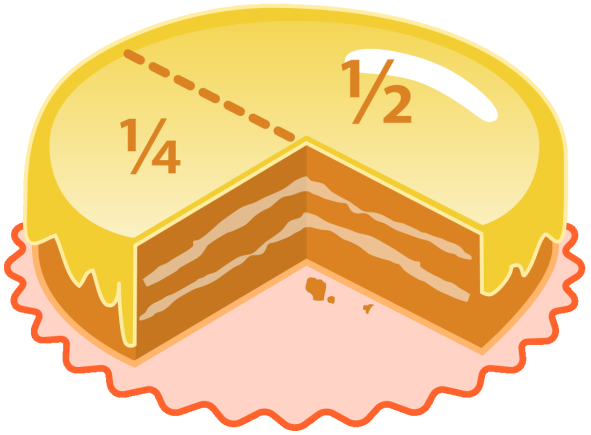
The first rule of addition is that only like quantities can be added; for example, various quantities of quarters. Unlike quantities, such as adding thirds to quarters, must first be converted to like quantities as described below: Imagine a pocket containing two quarters, and another pocket containing three quarters; in total, there are five quarters. Since four quarters is equivalent to one (dollar), this can be represented as follows:

$$\tfrac{2}{4} + \tfrac{3}{4} = \tfrac{5}{4} = 1\tfrac{1}{4}.$$

Adding unlike quantities

To add fractions containing unlike quantities (e.g. quarters and thirds), it is necessary to convert all amounts to like quantities. It is easy to work out the chosen type of fraction to convert to; simply multiply together the two denominators (bottom number) of each fraction.

For adding quarters to thirds, both types of fraction are converted to twelfths, thus: $\tfrac{1}{4} + \tfrac{1}{3} = \tfrac{1*3}{4*3} + \tfrac{1*4}{3*4} = \tfrac{3}{12} + \tfrac{4}{12} = \tfrac{7}{12}$.



If $\tfrac{1}{2}$ of a cake is to be added to $\tfrac{1}{4}$ of a cake, the pieces need to be converted into comparable quantities, such as cake-eighths or cake-quarters.

Consider adding the following two quantities:

$$\tfrac{3}{5} + \tfrac{2}{3}$$

First, convert $\tfrac{3}{5}$ into fifteenths by multiplying both the numerator and denominator by three: $\tfrac{3}{5} \times \tfrac{3}{3} = \tfrac{9}{15}$. Since $\tfrac{3}{3}$ equals 1, multiplication by $\tfrac{3}{3}$ does not change the value of the fraction.

Second, convert $\tfrac{2}{3}$ into fifteenths by multiplying both the numerator and denominator by five: $\tfrac{2}{3} \times \tfrac{5}{5} = \tfrac{10}{15}$.

Now it can be seen that:

$$\tfrac{3}{5} + \tfrac{2}{3}$$

is equivalent to:

$$\tfrac{9}{15} + \tfrac{10}{15} = \tfrac{19}{15} = 1\tfrac{4}{15}$$

This method can be expressed algebraically:

$$\tfrac{a}{b} + \tfrac{c}{d} = \tfrac{ad+cb}{bd}$$

And for expressions consisting of the addition of three fractions:

$$\tfrac{a}{b} + \tfrac{c}{d} + \tfrac{e}{f} = \tfrac{a(df)+c(bf)+e(bd)}{bdf}$$

This method always works, but sometimes there is a smaller denominator that can be used (a least common denominator). For example, to add $\tfrac{3}{4}$ and $\tfrac{5}{12}$ the denominator 48 can be used (the product of 4 and 12), but the smaller denominator 12 may also be used, being the least common multiple of 4 and 12.

$$\tfrac{3}{4} + \tfrac{5}{12} = \tfrac{9}{12} + \tfrac{5}{12} = \tfrac{14}{12} = \tfrac{7}{6} = 1\tfrac{1}{6}$$

Subtraction

The process for subtracting fractions is, in essence, the same as that of adding them: find a common denominator, and change each fraction to an equivalent fraction with the chosen common denominator. The resulting fraction will have that denominator, and its numerator will be the result of subtracting the numerators of the original fractions. For instance,

$$\frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$$

Multiplication

Multiplying a fraction by another fraction

To multiply fractions, multiply the numerators and multiply the denominators. Thus:

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$$

Why does this work? First, consider one third of one quarter. Using the example of a cake, if three small slices of equal size make up a quarter, and four quarters make up a whole, twelve of these small, equal slices make up a whole. Therefore a third of a quarter is a twelfth. Now consider the numerators. The first fraction, two thirds, is twice as large as one third. Since one third of a quarter is one twelfth, two thirds of a quarter is two twelfth. The second fraction, three quarters, is three times as large as one quarter, so two thirds of three quarters is three times as large as two thirds of one quarter. Thus two thirds times three quarters is six twelfths.

A short cut for multiplying fractions is called "cancellation". In effect, we reduce the answer to lowest terms during multiplication. For example:

$$\frac{2}{3} \times \frac{3}{4} = \frac{\cancel{2}^{\,1}}{\cancel{3}^{\,1}} \times \frac{\cancel{3}^{\,1}}{\cancel{4}^{\,2}} = \frac{1}{1} \times \frac{1}{2} = \frac{1}{2}$$

A two is a common factor in both the numerator of the left fraction and the denominator of the right and is divided out of both. Three is a common factor of the left denominator and right numerator and is divided out of both.

Multiplying a fraction by a whole number

Place the whole number over one and multiply.

$$6 \times \frac{3}{4} = \frac{6}{1} \times \frac{3}{4} = \frac{18}{4}$$

This method works because the fraction 6/1 means six equal parts, each one of which is a whole.

Mixed numbers

When multiplying mixed numbers, it's best to convert the mixed number into an improper fraction. For example:

$$3 \times 2\tfrac{3}{4} = 3 \times \left(\tfrac{8}{4} + \tfrac{3}{4}\right) = 3 \times \tfrac{11}{4} = \tfrac{33}{4} = 8\tfrac{1}{4}$$

In other words, $2\frac{3}{4}$is the same as $\frac{8}{4} + \frac{3}{4}$, making 11 quarters in total (because 2 cakes, each split into quarters makes 8 quarters total) and 33 quarters is $8\frac{1}{4}$, since 8 cakes, each made of quarters, is 32 quarters in total.

Division

To divide a fraction by a whole number, you may either divide the numerator by the number, if it goes evenly into the numerator, or multiply the denominator by the number. For example, $\frac{10}{3} \div 5$ equals $\frac{2}{3}$ and also equals $\frac{10}{3 \cdot 5} = \frac{10}{15}$, which reduces to $\frac{2}{3}$. To divide a number by a fraction, multiply that number by the reciprocal of that fraction. Thus, $\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{1 \cdot 4}{2 \cdot 3} = \frac{2}{3}$.

Converting between decimals and fractions

To change a common fraction to a decimal, divide the denominator into the numerator. Round the answer to the desired accuracy. For example, to change 1/4 to a decimal, divide 4 into 1.00, to obtain 0.25. To change 1/3 to a decimal, divide 3 into 1.0000..., and stop when the desired accuracy is obtained. Note that 1/4 can be written exactly with two decimal digits, while 1/3 cannot be written exactly with any finite number of decimal digits.

To change a decimal to a fraction, write in the denominator a 1 followed by as many zeroes as there are digits to the right of the decimal point, and write in the numerator all the digits in the original decimal, omitting the decimal point. Thus 12.3456 = 123456/10000.

Converting repeating decimals to fractions

Decimal numbers, while arguably more useful to work with when performing calculations, sometimes lack the precision that common fractions have. Sometimes an infinite repeating decimal is required to reach the same precision. Thus, it is often useful to convert repeating decimals into fractions.

The preferred way to indicate a repeating decimal is to place a bar over the digits that repeat, for example 0.789 = 0.789789789… For repeating patterns where the repeating pattern begins immediately after the decimal point, a simple division of the pattern by the same number of nines as numbers it has will suffice. For example:

0.5 = 5/9

0.62 = 62/99

0.264 = 264/999

0.6291 = 6291/9999

In case leading zeros precede the pattern, the nines are suffixed by the same number of trailing zeros:

0.05 = 5/90

0.000392 = 392/999000

0.0012 = 12/9900

In case a non-repeating set of decimals precede the pattern (such as 0.1523987), we can write it as the sum of the non-repeating and repeating parts, respectively:

0.1523 + 0.0000987

Then, convert both parts to fractions, and add them using the methods described above:

1523/10000 + 987/9990000 = 1522464/9990000

Fractions in abstract mathematics

In addition to being of great practical importance, fractions are also studied by mathematicians, who check that the rules for fractions given above are consistent and reliable. Mathematicians define a fraction as an ordered pair (*a*, *b*) of integers *a* and *b* ≠ 0, for which the operations addition, subtraction, multiplication, and division are defined as follows:

$$(a, b) + (c, d) = (ad + bc, bd)$$
$$(a, b) - (c, d) = (ad - bc, bd)$$
$$(a, b) \cdot (c, d) = (ac, bd)$$
$$(a, b) \div (c, d) = (ad, bc) \quad \text{(when c} \neq 0\text{)}$$

In addition, an equivalence relation is specified as follows: $(a, b) \sim (c, d)$ if and only if $ad = bc$.

These definitions agree in every case with the definitions given above; only the notation is different.

More generally, *a* and *b* may be elements of any integral domain *R*, in which case a fraction is an element of the field of fractions of *R*. For example, when *a* and *b* are polynomials in one indeterminate, the field of fractions is the field of rational fractions (also known as the field of rational functions). When *a* and *b* are integers, the field of fractions is the field of rational numbers.

Algebraic fractions

An algebraic fraction is the indicated quotient of two algebraic expressions. Two examples of algebraic fractions are $\frac{3x}{x^2 + 2x - 3}$ and $\frac{\sqrt{x+2}}{x^2 - 3}$. Algebraic fractions are subject to the same laws as arithmetic fractions.

If the numerator and the denominator are polynomials, as in $\frac{3x}{x^2 + 2x - 3}$, the algebraic fraction is called a **rational fraction** (or **rational expression**). An **irrational fraction** is one that contains the variable under a fractional exponent or root, as in $\frac{\sqrt{x+2}}{x^2 - 3}$.

The terminology used to describe algebraic fractions is similar to that used for ordinary fractions. For example, an algebraic fraction is in lowest terms if the only factors common to the numerator and the denominator are 1 and −1. An algebraic fraction whose numerator or denominator, or both, contain a fraction, such as $\frac{1 + \frac{1}{x}}{1 - \frac{1}{x}}$, is called a **complex fraction**.

Rational numbers are the quotient field of integers. Rational expressions are the quotient field of the polynomials (over some integral domain). Since a coefficient is a polynomial of degree zero, a radical expression such as √2/2 is a rational fraction. Another example (over the reals) is $\frac{\pi}{2}$, the radian measure of a right angle.

The term partial fraction is used when decomposing rational expressions into sums. The goal is to write the rational expression as the sum of other rational expressions with denominators of lesser degree. For example, the rational expression $\frac{2x}{x^2-1}$ can be rewritten as the sum of two fractions: $\frac{1}{x+1} + \frac{1}{x-1}$. This is useful in many areas such as integral calculus and differential equations.

Radical expressions

A fraction may also contain radicals in the numerator and/or the denominator. If the denominator contains radicals, it can be helpful to rationalize it (compare Simplified form of a radical expression), especially if further operations, such as adding or comparing that fraction to another, are to be carried out. It is also more convenient if division is to be done manually. When the denominator is a monomial square root, it can be rationalized by multiplying both the top and the bottom of the fraction by the denominator:

$$\frac{3}{\sqrt{7}} = \frac{3}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$$

The process of rationalization of binomial denominators involves multiplying the top and the bottom of a fraction by the conjugate of the denominator so that the denominator becomes a rational number. For example:

$$\frac{3}{3-2\sqrt{5}} = \frac{3}{3-2\sqrt{5}} \cdot \frac{3+2\sqrt{5}}{3+2\sqrt{5}} = \frac{3(3+2\sqrt{5})}{3^2-(2\sqrt{5})^2} = \frac{3(3+2\sqrt{5})}{9-20} = -\frac{9+6\sqrt{5}}{11}$$

$$\frac{3}{3+2\sqrt{5}} = \frac{3}{3+2\sqrt{5}} \cdot \frac{3-2\sqrt{5}}{3-2\sqrt{5}} = \frac{3(3-2\sqrt{5})}{3^2-(2\sqrt{5})^2} = \frac{3(3-2\sqrt{5})}{9-20} = -\frac{9-6\sqrt{5}}{11}$$

Even if this process results in the numerator being irrational, like in the examples above, the process may still facilitate subsequent manipulations by reducing the number of irrationals one has to work with in the denominator.

Typographical variations

In computer displays and typography, simple fractions are sometimes printed as a single character, e.g. ½ (one half). See the article on Number Forms for information on doing this in Unicode.

Scientific publishing distinguishes four ways to set fractions, together with guidelines on use:

- **special fractions:** fractions that are presented as a single character with a slanted bar, with roughly the same height and width as other characters in the text. Generally used for simple fractions, such as: ½, ⅓, ⅔, ¼, and ¾. Since the numerals are smaller, legibility can be an issue, especially for small-sized fonts. These are not used in modern mathematical notation, but in other contexts.
- **case fractions:** similar to special fractions, these are rendered as a single typographical character, but with a horizontal bar, thus making them *upright*. An example would be $\frac{1}{2}$, but rendered with the same height as other characters. Some sources include all rendering of fractions as *case fractions* if they take only one typographical space, regardless of the direction of the bar.
- **shilling fractions:** 1/2, so called because this notation was used for pre-decimal British currency (£sd), as in 2/6 for a half crown, meaning two shillings and six pence. While the notation "two shillings and six pence" did not represent a fraction, the forward slash is now used in fractions, especially for fractions inline with prose (rather than displayed), to avoid uneven lines. It is also used for fractions within fractions (complex fractions) or within exponents to increase legibility. Fractions written this way, also known as *piece fractions*, are written all on one typographical line, but take 3 or more typographical spaces.
- **built-up fractions:** $\frac{1}{2}$. This notation uses two or more lines of ordinary text, and results in a variation in spacing between lines when included within other text. While large and legible, these can be disruptive, particularly for simple fractions or within complex fractions.

History

The earliest fractions were reciprocals of integers: ancient symbols representing one part of two, one part of three, one part of four, and so on. The Egyptians used Egyptian fractions ca. 1000 BC. About 4,000 years ago Egyptians divided with fractions using slightly different methods. They used least common multiples with unit fractions. Their methods gave the same answer as modern methods.[5] The Egyptians also had a different notation for dyadic fractions in the Akhmim Wooden Tablet and several Rhind Mathematical Papyrus problems.

The Greeks used unit fractions and later continued fractions and followers of the Greek philosopher Pythagoras, ca. 530 BC, discovered that the square root of two cannot be expressed as a fraction. In 150 BC Jain mathematicians in India wrote the "Sthananga Sutra", which contains work on the theory of numbers, arithmetical operations, operations with fractions.

The method of putting one number below the other and computing fractions first appeared in Aryabhatta's work around AD 499.[*citation needed*] In Sanskrit literature, fractions, or rational numbers were always expressed by an integer followed by a fraction. When the integer is written on a line, the fraction is placed below it and is itself written on two lines, the numerator called *amsa* part on the first line, the denominator called *cheda* “divisor” on the second below. If the fraction is written without any particular additional sign, one understands that it is added to the integer above it. If it is marked by a small circle or a cross (the shape of the “plus” sign in the West) placed on its right, one understands that it is subtracted from the integer. For example (to be read vertically), Bhaskara I writes

६	१	२
१	१	१०
४	५	९

That is,

6	1	2
1	1	$1_{\circ}$
4	5	9

to denote 6+1/4, 1+1/5, and 2−1/9.

Al-Hassār, a Muslim mathematician from Fez, Morocco specializing in Islamic inheritance jurisprudence during the 12th century, first mentions the use of a fractional bar, where numerators and denominators are separated by a horizontal bar. In his discussion he writes, "... for example, if you are told to write three-fifths and a third of a fifth, write thus, $\frac{3 \quad 1}{5 \quad 3}$." This same fractional notation appears soon after in the work of Leonardo Fibonacci in the 13th century.

In discussing the origins of decimal fractions, Dirk Jan Struik states:

> "The introduction of decimal fractions as a common computational practice can be dated back to the Flemish pamphlet *De Thiende*, published at Leyden in 1585, together with a French translation, *La Disme*, by the Flemish mathematician Simon Stevin (1548–1620), then settled in the Northern Netherlands. It is true that decimal fractions were used by the Chinese many centuries before Stevin and that the Persian astronomer Al-Kāshī used both decimal and sexagesimal fractions with great ease in his *Key to arithmetic* (Samarkand, early fifteenth century)."

While the Persian mathematician Jamshīd al-Kāshī claimed to have discovered decimal fractions himself in the 15th century, J. Lennart Berggren notes that he was mistaken, as decimal fractions were first used five centuries before him by the Baghdadi mathematician Abu'l-Hasan al-Uqlidisi as early as the 10th century.[6]

In formal education

Pedagogical tools

In primary schools, fractions have been demonstrated through Cuisenaire rods, Fraction Bars, fraction strips, fraction circles, paper (for folding or cutting), pattern blocks, pie-shaped pieces, plastic rectangles, grid paper, dot paper, geoboards, counters and computer software.

Documents for teachers

Several states in the United States have adopted learning trajectories from the Common Core State Standards Initiative's guidelines for mathematics education. Aside from sequencing the learning of fractions and operations with fractions, the document provides the following definition of a fraction: "A number expressible in the form $\frac{a}{b}$ where a is a whole number and b is a positive whole number. (The word *fraction* in the standards always refers to a non-negative number.)" The document itself also refers to negative fractions.

References

[1] H. Wu, *The Mis-Education of Mathematics Teachers*, Notices of the American Mathematical Society, Volume 58, Issue 03 (March 2011), page 374 (http://www.ams.org/notices/201103/rtx110300372p.pdf#page374)

[2] World Wide Words: Vulgar fractions (http://www.worldwidewords.org/qa/qa-vul1.htm)

[3] Math Forum – Ask Dr. Math:Can Negative Fractions Also Be Proper or Improper? (http://mathforum.org/library/drmath/view/65128.html)

[4] New England Compact Math Resources (http://www.necompact.org/ea/gle_support/Math/resources_number/prop_fraction.htm)

[5] See for examples and an explanation.

[6] While there is some disagreement among history of mathematics scholars as to the primacy of al-Uqlidisi's contribution, there is no question as to his major contribution to the concept of decimal fractions. (http://www-history.mcs.st-andrews.ac.uk/Biographies/Al-Uqlidisi.html) "MacTutor's al-Uqlidisi biography". Retrieved 2011-11-22.

External links

- "Fraction, arithmetical" (http://www.encyclopediaofmath.org/index.php/Fraction). *The Online Encyclopaedia of Mathematics*.
- Weisstein, Eric W., " Fraction (http://mathworld.wolfram.com/Fraction.html)", *MathWorld*.
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- "Fraction (mathematics)" (http://en.citizendium.org/wiki/Fraction_(mathematics)). *Citizendium*.
- "Fraction" (http://planetmath.org/encyclopedia/Fraction.html). *PlanetMath*.
- Online program (http://dl5jaf.darc.de/math/rationotation/) for exact conversion between fractions and decimals

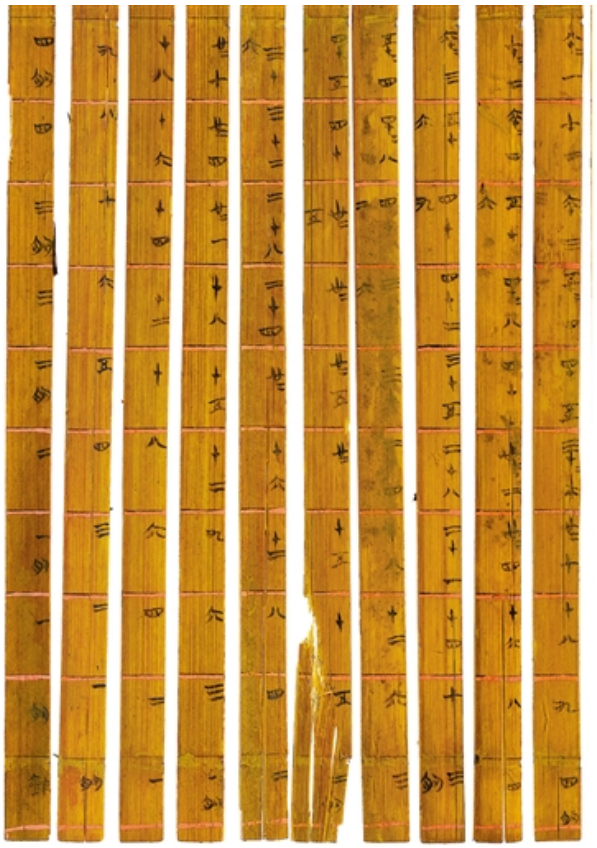
Decimal

This article aims to be an accessible introduction. For the mathematical definition, see Decimal representation.

The **decimal** numeral system (also called **base ten** or occasionally **denary**) has ten as its base. It is the numerical base most widely used by modern civilizations.[1][2]

Decimal notation often refers to a base-10 positional notation such as the Hindu-Arabic numeral system; however, it can also be used more generally to refer to non-positional systems such as Roman or Chinese numerals which are also based on powers of ten.

Decimals also refer to decimal fractions, either separately or in contrast to vulgar fractions. In this context, a decimal is a tenth part, and decimals become a series of nested tenths. There was a notation in use like 'tenth-metre', meaning the tenth decimal of the metre, currently an Angstrom. The contrast here is between decimals and vulgar fractions, and decimal divisions and other divisions of measures, like the inch. It is possible to follow a decimal expansion with a vulgar fraction; this is done with the recent divisions of the troy ounce, which has three places of decimals, followed by a trinary place.



The world's earliest decimal multiplication table was made from bamboo slips, dating from 305 BC, during the Warring States period

Decimal notation

Decimal notation is the writing of numbers in a base-10 numeral system. Examples are Roman numerals, Brahmi numerals, and Chinese numerals, as well as the Hindu-Arabic numerals used by speakers of many European languages. Roman numerals have symbols for the decimal powers (1, 10, 100, 1000) and secondary symbols for half these values (5, 50, 500). Brahmi numerals have symbols for the nine numbers 1–9, the nine decades 10–90, plus a symbol for 100 and another for 1000. Chinese numerals have symbols for 1–9, and additional symbols for powers of 10, which in modern usage reach 10^{44}.

However, when people who use Hindu-Arabic numerals speak of decimal notation, they often mean not just decimal numeration, as above, but also decimal fractions, all conveyed as part of a positional system. Positional decimal systems include a zero and use symbols (called digits) for the ten values (0, 1, 2, 3, 4, 5, 6, 7, 8, and 9) to represent any number, no matter how large or how small. These digits are often used with a decimal separator which indicates the start of a fractional part, and with a symbol such as the plus sign + (for positive) or minus sign − (for negative) adjacent to the numeral to indicate whether it is greater or less than zero, respectively.

Positional notation uses positions for each power of ten: units, tens, hundreds, thousands, etc. The position of each digit within a number denotes the multiplier (power of ten) multiplied with that digit—each position has a value ten

times that of the position to its right. There were at least two presumably independent sources of positional decimal systems in ancient civilization: the Chinese counting rod system and the Hindu-Arabic numeral system (the latter descended from Brahmi numerals).

Ten is the number which is the count of fingers and thumbs on both hands (or toes on the feet). The English word digit as well as its translation in many languages is also the anatomical term for fingers and toes. In English, decimal (decimus < Lat.) means *tenth*, decimate means *reduce by a tenth*, and denary (denarius < Lat.) means *the unit of ten*.

The symbols for the digits in common use around the globe today are called Arabic numerals by Europeans and Indian numerals by Arabs, the two groups' terms both referring to the culture from which they learned the system. However, the symbols used in different areas are not identical; for instance, Western Arabic numerals (from which the European numerals are derived) differ from the forms used by other Arab cultures.

Decimal fractions

A **decimal fraction** is a fraction whose denominator is a power of ten.

Decimal fractions are commonly expressed without a denominator, the decimal separator being inserted into the numerator (with leading zeros added if needed) at the position from the right corresponding to the power of ten of the denominator; e.g., 8/10, 83/100, 83/1000, and 8/10000 are expressed as 0.8, 0.83, 0.083, and 0.0008. In English-speaking, some Latin American and many Asian countries, a period (**.**) or raised period (·) is used as the decimal separator; in many other countries, particularly in Europe, a comma is used.

The integer part, or integral part of a decimal number is the part to the left of the decimal separator. (See also truncation.) The part from the decimal separator to the right is the *fractional part*. It is usual for a decimal number that consists only of a fractional part (mathematically, a *proper fraction*) to have a leading zero in its notation (its *numeral*). This helps disambiguation between a decimal sign and other punctuation, and especially when the negative number sign is indicated, it helps visualize the sign of the numeral as a whole.

Trailing zeros after the decimal point are not necessary, although in science, engineering and statistics they can be retained to indicate a required precision or to show a level of confidence in the accuracy of the number: Although 0.080 and 0.08 are numerically equal, in engineering 0.080 suggests a measurement with an error of up to one part in two thousand (±0.0005), while 0.08 suggests a measurement with an error of up to one in two hundred (see *significant figures*).

Other rational numbers

Any rational number with a denominator whose only prime factors are 2 and/or 5 may be precisely expressed as a decimal fraction and has a finite decimal expansion.

1/2 = 0.5

1/20 = 0.05

1/5 = 0.2

1/50 = 0.02

1/4 = 0.25

1/40 = 0.025

1/25 = 0.04

1/8 = 0.125

1/125 = 0.008

1/10 = 0.1

If the rational number's denominator has any prime factors other than 2 or 5, it cannot be expressed as a finite decimal fraction, and has a unique eventually repeating infinite decimal expansion.

1/3 = 0.333333… (with 3 repeating)

1/9 = 0.111111… (with 1 repeating)

100 − 1 = 99 = 9 × 11:

1/11 = 0.090909…

1000 − 1 = 9 × 111 = 27 × 37:

1/27 = 0.037037037…

1/37 = 0.027027027…

1/111 = 0 .009009009…

also:

1/81 = 0.012345679012… (with 012345679 repeating)

That a rational number must have a finite or recurring decimal expansion can be seen to be a consequence of the long division algorithm, in that there are only q-1 possible nonzero remainders on division by q, so that the recurring pattern will have a period less than q. For instance, to find 3/7 by long division:

```
   0.4 2 8 5 7 1 4 ...
7 ) 3.0 0 0 0 0 0 0 0
    2 8                          30/7 = 4 with a remainder of 2
      2 0
      1 4                        20/7 = 2 with a remainder of 6
        6 0
        5 6                      60/7 = 8 with a remainder of 4
          4 0
          3 5                    40/7 = 5 with a remainder of 5
            5 0
            4 9                  50/7 = 7 with a remainder of 1
              1 0
              7                  10/7 = 1 with a remainder of 3
                3 0
                2 8              30/7 = 4 with a remainder of 2
                  2 0
                       etc.
```

The converse to this observation is that every recurring decimal represents a rational number *p*/*q*. This is a consequence of the fact that the recurring part of a decimal representation is, in fact, an infinite geometric series which will sum to a rational number. For instance,

$$0.0123123123\cdots = \frac{123}{10000}\sum_{k=0}^{\infty} 0.001^k = \frac{123}{10000}\ \frac{1}{1-0.001} = \frac{123}{9990} = \frac{41}{3330}$$

Real numbers

Every real number has a (possibly infinite) decimal representation; i.e., it can be written as

$$x = \operatorname{sign}(x) \sum_{i \in \mathbb{Z}} a_i \, 10^i$$

where

- sign() is the sign function,
- **Z** is the set of all integers (positive, negative, and zero), and
- $a_i \in \{ 0,1,\ldots,9 \}$ for all $i \in \mathbf{Z}$ are its **decimal digits**, equal to zero for all i greater than some number (that number being the common logarithm of |x|).

Such a sum converges as more and more negative values of i are included, even if there are infinitely many non-zero a_i.

Rational numbers (e.g., p/q) with prime factors in the denominator other than 2 and 5 (when reduced to simplest terms) have a unique recurring decimal representation.

Non-uniqueness of decimal representation

Consider those rational numbers which have only the factors 2 and 5 in the denominator, i.e., which can be written as $p/(2^a 5^b)$. In this case there is a terminating decimal representation. For instance, 1/1 = 1, 1/2 = 0.5, 3/5 = 0.6, 3/25 = 0.12 and 1306/1250 = 1.0448. Such numbers are the only real numbers which do not have a unique decimal representation, as they can also be written as a representation that has a recurring 9, for instance 1 = 0.99999…, 1/2 = 0.499999…, etc. The number 0 = 0/1 is special in that it has no representation with recurring 9.

This leaves the irrational numbers. They also have unique infinite decimal representations, and can be characterised as the numbers whose decimal representations neither terminate nor recur.

So in general the decimal representation is unique, if one excludes representations that end in a recurring 9.

The same trichotomy holds for other base-n positional numeral systems:

- Terminating representation: rational where the denominator divides some n^k
- Recurring representation: other rational
- Non-terminating, non-recurring representation: irrational

A version of this even holds for irrational-base numeration systems, such as golden mean base representation.

Decimal computation

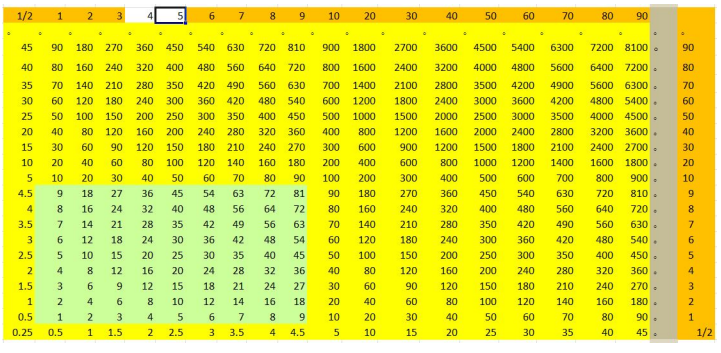


Diagram of the world's earliest decimal multiplication table (c. 305 BC) from the Warring States period

Decimal computation was carried out in ancient times in many ways, typically in rod calculus, with decimal multiplication table used in ancient China and with sand tables in India and Middle East or with a variety of abaci.

Modern computer hardware and software systems commonly use a binary representation internally (although many early computers, such as the ENIAC or the IBM 650, used decimal representation internally).[3] For external use by computer specialists, this binary representation is sometimes presented in the related octal or hexadecimal systems.

For most purposes, however, binary values are converted to or from the equivalent decimal values for presentation to or input from humans; computer programs express literals in decimal by default. (123.1, for example, is written as such in a computer program, even though many computer languages are unable to encode that number precisely.)

Both computer hardware and software also use internal representations which are effectively decimal for storing decimal values and doing arithmetic. Often this arithmetic is done on data which are encoded using some variant of binary-coded decimal,[4] especially in database implementations, but there are other decimal representations in use (such as in the new IEEE 754 Standard for Floating-Point Arithmetic).[5]

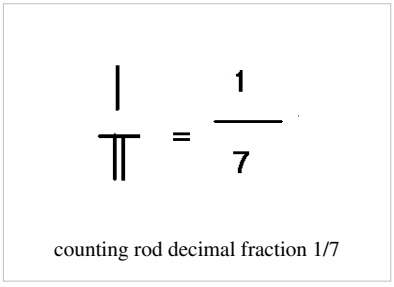
Decimal arithmetic is used in computers so that decimal fractional results can be computed exactly, which is not possible using a binary fractional representation. This is often important for financial and other calculations.[6]

History

Many ancient cultures calculated from early on with numerals based on ten: Egyptian hieroglyphs, in evidence since around 3000 BC, used a purely decimal system,[7][8] just as the Cretan hieroglyphs (ca. 1625–1500 BC) of the Minoans whose numerals are closely based on the Egyptian model.[9][10] The decimal system was handed down to the consecutive Bronze Age cultures of Greece, including Linear A (ca. 18th century BC–1450 BC) and Linear B (ca. 1375–1200 BC) — the number system of classical Greece also used powers of ten, including, like the Roman numerals did, an intermediate base of 5.[11] Notably, the polymath Archimedes (c. 287–212 BC) invented a decimal positional system in his Sand Reckoner which was based on 10^8 and later led the German mathematician Carl Friedrich Gauss to lament what heights science would have already reached in his days if Archimedes had fully realized the potential of his ingenious discovery.[12] The Hittites hieroglyphs (since 15th century BC), just like the Egyptian and early numerals in Greece, was strictly decimal.[13]

The Egyptian hieratic numerals, the Greek alphabet numerals, the Roman numerals, the Chinese numerals and early Indian Brahmi numerals are all non-positional decimal systems, and required large numbers of symbols. For instance, Egyptian numerals used different symbols for 10, 20, to 90, 100, 200, to 900, 1000, 2000, 3000, 4000, to 10,000.[14]

History of decimal fractions



counting rod decimal fraction 1/7

According to Joseph Needham, decimal fractions were first developed and used by the Chinese in the 1st century BC, and then spread to the Middle East and from there to Europe. The written Chinese decimal fractions were non-positional. However, counting rod fractions were positional.

Qin Jiushao in his book Mathematical Treatise in Nine Sections (1247) denoted 0.96644 by

寸


, meaning

寸

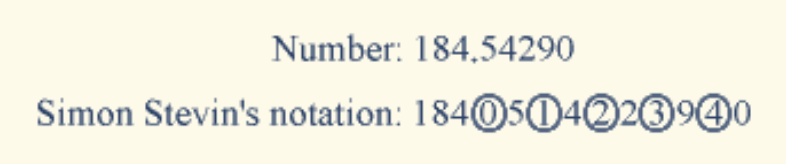
096644

[15]

The Jewish mathematician Immanuel Bonfils invented decimal fractions around 1350, anticipating Simon Stevin, but did not develop any notation to represent them.[16]

The Persian mathematician Jamshīd al-Kāshī claimed to have discovered decimal fractions himself in the 15th century, though J. Lennart Berggren notes that positional decimal fractions were used five centuries before him by Arab mathematician Abu'l-Hasan al-Uqlidisi as early as the 10th century.

Khwarizmi introduced fractions to Islamic countries in the early 9th century. . This form of fraction with the numerator on top and the denominator on the bottom, without a horizontal bar, was also used in the 10th century by Abu'l-Hasan al-Uqlidisi and again in the 15th century work "Arithmetic Key" by Jamshīd al-Kāshī.[*citation needed*]



A forerunner of modern European decimal notation was introduced by Simon Stevin in the 16th century.

Natural languages

Telugu language uses a straightforward decimal system. Other Dravidian languages such as Tamil and Malayalam have replaced the number nine *tondu* with 'onpattu' ("one to ten") during the early Middle Ages, while Telugu preserved the number nine as *tommidi*.

The Hungarian language also uses a straightforward decimal system. All numbers between 10 and 20 are formed regularly (e.g. 11 is expressed as "tízenegy" literally "one on ten"), as with those between 20-100 (23 as "huszonhárom" = "three on twenty").

A straightforward decimal rank system with a word for each order 10十,100百,1000千,10000万, and in which 11 is expressed as *ten-one* and 23 as *two-ten-three*, and 89345 is expressed as 8 (ten thousands) 万9 (thousand) 千3 (hundred) 百4 (tens) 十 5 is found in Chinese languages, and in Vietnamese with a few irregularities. Japanese, Korean, and Thai have imported the Chinese decimal system. Many other languages with a decimal system have special words for the numbers between 10 and 20, and decades. For example in English 11 is "eleven" not "ten-one".

Incan languages such as Quechua and Aymara have an almost straightforward decimal system, in which 11 is expressed as *ten with one* and 23 as *two-ten with three*.

Some psychologists suggest irregularities of the English names of numerals may hinder children's counting ability.

Other bases

Some cultures do, or did, use other bases of numbers.

- Pre-Columbian Mesoamerican cultures such as the Maya used a base-20 system (using all twenty fingers and toes).
- The Yuki language in California and the Pamean languages in Mexico have octal (base-8) systems because the speakers count using the spaces between their fingers rather than the fingers themselves.
- The existence of a non-decimal base in the earliest traces of the Germanic languages, is attested by the presence of words and glosses meaning that the count is in decimal (cognates to ten-count or tenty-wise), such would be expected if normal counting is not decimal, and unusual if it were.Wikipedia:No original research#Synthesis of published material that advances a position Where this counting system is known, it is based on the long hundred of 120 in number, and a long thousand of 1200 in number. The descriptions like 'long' only appear after the small hundred of 100 in number appeared with the Christians. Gordon's Introduction to Old Norse [17] p 293, gives number names that belong to this system. An expression cognate to 'one hundred and eighty' is translated to 200, and the cognate to 'two hundred' is translated at 240. Goodare [18] details the use of the long hundred in Scotland in the Middle Ages, giving examples, calculations where the carry implies i C (i.e. one hundred) as 120, etc. That the general population were not alarmed to encounter such numbers suggests common enough use. It is also possible to avoid hundred-like numbers by using intermediate units, such as stones and pounds, rather than a long count of pounds. Goodare gives examples of numbers like vii score, where one avoids the hundred by using extended scores. There is also a paper by W.H. Stevenson, on 'Long Hundred and its uses in England'. [*citation needed*]

- Many or all of the Chumashan languages originally used a base-4 counting system, in which the names for numbers were structured according to multiples of 4 and 16.[19]
- Many languages[20] use quinary (base-5) number systems, including Gumatj, Nunggubuyu, Kuurn Kopan Noot[21] and Saraveca. Of these, Gumatj is the only true 5–25 language known, in which 25 is the higher group of 5.
- Some Nigerians use base-12 systems[*citation needed*]
- The Huli language of Papua New Guinea is reported to have base-15 numbers. *Ngui* means 15, *ngui ki* means 15×2 = 30, and *ngui ngui* means 15×15 = 225.
- Umbu-Ungu, also known as Kakoli, is reported to have base-24 numbers. *Tokapu* means 24, *tokapu talu* means 24×2 = 48, and *tokapu tokapu* means 24×24 = 576.
- Ngiti is reported to have a base-32 number system with base-4 cycles.

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External links

- Decimal arithmetic FAQ (http://speleotrove.com/decimal/decifaq.html)
- Cultural Aspects of Young Children's Mathematics Knowledge (http://spot.colorado.edu/~gubermas/NCTM_pap.htm)

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